https://doi.org/10.17221/134/2019-AGRICECON

Optimal treatment of agricultural land – special multi-depot vehicle routing problem

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Citation: Andric Gusavac B., Stanojevic M., Cangalovic M. (2019): Optimal treatment of agricultural land – special multi-depot vehicle routing problem. Agricultural Economics – Czech, 65: 569–578.

Abstract: This paper describes a problem of optimal agricultural land treatment using aviation. The studied problem consists of determining the optimal routes for a given set of aircraft used for chemical treatment of arable agricultural land divided into parcels. This NP (nondeterministic polynomial time) problem is represented on a graph and a mixed integer mathematical programming model of the problem is formulated. This mathematical model is a specific variant of the multi-depot vehicle routing problem where a min-cost plan for the transportation of a homogeneous product (chemicals used for land treatment) from different supply locations (airfields) to different demand locations (agricultural parcels) should be generated. Some specifics of the agricultural land chemical treatment are described in the paper and the following specific conditions are taken into consideration: each parcel is treated only by one way of treatment and one aircraft; for each aircraft its chemical and fuel reservoir capacities are sufficient to serve its route. The complexity of the problem and the impossibility to obtain exact solutions for larger dimensions of the problem led to the formulation of a special heuristics which is presented in this paper. Numerical experiments are successfully conducted for larger problem dimensions and results are presented.

Keywords: agriculture; combinatorial optimisation; mathematical model; multi-depot vehicle routing problem; optimisation; special heuristics

Application of operations research models and methods has become important in assisting farmers and extension specialists decide whether to introduce alternative practices to make agriculture more sustainable (Hayashi 2000). Today, due to extensive use, quality of the arable land decreases and it is getting more and more polluted. The review of available literature proved that in the field of agriculture different operations research problems, location problems, problem of allocation and routing are being solved. Problems of determining the optimal routes for agricultural vehicles (including robots) on agricultural land are often solved, by genetic algorithms (Gracia et al. 2014; Mahmud et al. 2018; Mahmud et al. 2019), by modified Clark-Wright algorithm (Seyyedhasani and Dvorak 2017) or implementing mix-opt metaheuristic operator (Conesa-Munoz et al. 2016). There are a number of examples in the literature where various agricultural problems are solved by optimisation: allocation of agricultural land and water resources (Mosleh et al. 2017); determining the order in which automated vehicles visit plots (blocks) of agricultural land (Hameed et al. 2013); fleet management of agricultural vehicles including resource allocation, scheduling, routing, real-time monitoring of vehicles and materials (Sorensen and Bochtis 2010); determining the location for agricultural airfields (Andrić Gušavac et al. 2014). Problems related to the agricultural land, its quality or arrangement of plots, are also being solved. Author

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in (Kung 2018) analyses how changes in the land fertility affect agricultural activities and bioenergy development, and authors in (Harasimowicz et al. 2017) deal with optimisation of land consolidation.

Agriculturalists are the principal managers of global usable lands and will shape, perhaps irreversibly, the surface of the Earth in the coming decades (Ewers et al. 2009). This will lead to the transformation of many landscapes from natural vegetation cover to agricultural land use, unless increases in crop yields reduce the need for new farmland.

Pesticide application in agriculture refers to the practical way in which pesticides (e.g. herbicides or fungicides) are delivered to their biological targets (e.g. crop or other plant) using agricultural vehicles or airplanes. Authors in (Mghirbi et al. 2017) propose a decision tool for management of plant protection in agriculture in order to reduce the risk of pesticide toxicity. Adequate pesticide application technologies can improve efficiency of its use and protect public health and the environment.

The objective of this research is to determine the optimal routes for a given set of aircraft used for chemical treatment of arable agricultural land divided into parcels. In order to achieve this goal, the original mixed integer mathematical programming model of the problem is formulated and solved. Some preliminary numerical experiments to obtain exact solutions are given, and due to the complexity of the problem, a special heuristics is formulated in order to solve larger dimensions of the problem. The contribution of this paper is to provide analytical approach to agricultural land treatment planning, by defining the specific characteristics of the observed problem, formulating optimisation model and solving it, all of which could assist the farmers in land treatment planning and decrease the costs of it.

MATERIALS AND METHODS

The paper focuses on the specific agriculture problem – chemical soil treatment where an agricultural land divided into parcels has to be treated with chemicals. In order to find an optimal treatment of agricultural land using aviation we introduce a special variant of the multi-depot vehicle routing problem (VRP). VRP consists of finding an optimal set of routes for a fleet of vehicles which must serve a given set of customers and is frequently used for modelling and solving various agricultural problems. It has been over fifty years since Dantzig and Ramser (1959) introduced this problem. Some preliminary numerical experiments to obtain exact solutions are given, and due to the complexity of the problem, a special heuristics is formulated in order to solve larger dimensions of the problem.

Description of optimal treatment of agricultural land using aviation

We will describe in more precise way the problem of an optimal treatment of agricultural land using aviation (OTALA problem): an agricultural land is divided into a given set of parcels which should be chemically treated by an agricultural aviation. Given are set of aircraft used to realise this treatment and set of airfields that can be used for their take-offs and landings. A set of possible ways how to treat one parcel by the aircraft is defined for each parcel and each aircraft.

Problem OTALA is defined in the following way: for each aircraft we determine an airfield where it starts and finishes its flight, and a route (sequence of parcels) which it should treat in such way that the total treatment cost is minimal and the following conditions are fulfilled:

– each parcel is treated only by one way of treatment and one aircraft;
– for each aircraft its chemical and fuel reservoir capacities are sufficient to serve its route and the flight between each two adjacent parcels in this route is technically possible.

In this paper, we consider the real life problem which has 245 parcels and 7 potential locations for the airfields. This problem is noticed in an agricultural company, and it has not been solved using mathematical methods.

One example of a land treatment using aviation with one airfield and two parcels is presented in Figure 1, where a way of treatment for each parcel is defined. The way of treatment for the succeeding parcel depends on the way of treatment of the preceding parcel. In Figure 1, two ways of treatment (with three tracks each) for these two parcels are intuitively selected.

In the problem which is considered in this paper, we will assume that, for each parcel and the aircraft which can treat the parcel, a set of possible ways of treatment is given in advance.

The most frequently used treatment of a parcel is depictured in Figure 1, for such treatment makes the least length in flight when plane is turning, and thus the processing costs are lower.

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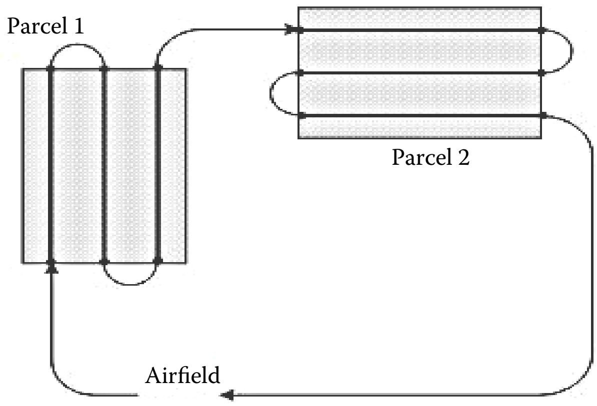

Figure 1. Two parcels treated in one route

Source: authors' composition

Based on Figure 1, we can notice two key points in the processing of the parcel – the point of entry of the aircraft and the point of its exit – when the entrance point is known, the exit point can be calculated based on the entry point and the possible number (odd or even) of tracks. The entry point and the exit point define one way of treatment for one parcel.

Representation of OTALA problem on graph

Problem OTALA can be represented as an optimisation problem on an associated weighted graph in the following way.

Let us introduce the following notations: given are a set of parcels – P, a set of aircraft – A and a set of airfields – L. For each $p \in P$ and each $a \in A$ a set of ways $V(p,a)$ to treat parcel p by aircraft a can be defined where $V(p,a)$ can be $\emptyset$.

The node-and-arc weighted graph now $G = (N,E,D,B)$ is associated to the problem OTALA as follows.

Set of nodes $N, |N| = |L| + \sum_{p \in P} \sum_{a \in A} |V(p,a)|$, is defined in the following way:

- each airfield $l \in L$ is represented by a node in G;
- for each parcel $p \in P$ and each aircraft $a \in A$ for which $V(p,a) \neq \emptyset$, each $v \in V(p,a)$ is represented as a node in G.

Set of arcs E is defined in the following way:

- for each airfield node $l \in L$ and each parcel $p \in P$ and each aircraft $a \in A$ for which $V(p,a) \neq \emptyset$, there are arcs (l,v) and (v,l) for each node $v \in V(p,a)$;
- for every two parcels $p_1, p_2 \in P$ and each aircraft $a \in A$ for which $V(p_1,a) \neq \emptyset$ and $V(p_2,a) \neq \emptyset$, there is an arc (v_1,v_2) for every $v_1 \in V(p_1,a)$ and $v_2 \in V(p_2,a)$ such that flight of aircraft a from parcel p_1 treated by way v_1 to parcel p_2 treated by way v_2 is technically possible (exit point of the treatment from the preceding parcel is connectable to entry point of the treatment for the succeeding parcel).

Set of node weights $D, D = \{d_v, v \in N\}$ is specified as:

- to each node $v \in N \setminus L$, corresponding to parcel $p \in P$ and aircraft $a \in A$ (i.e. $v \in V(p,a)$), a positive real weight d_v is associated as the length of flight of aircraft a over the parcel p which is treated by way v (in units of length);
- to each node $v \in L$, $d_v = 0$.

Set of arc weights $B, B = \{b_{vw}, (v,w) \in E\}$, is specified as:

- to each arc $(v,w) \in E$, where $v \in L$ and $w \in V(p,a)$, a positive real weight b_{vw} is associated as the length of flight (in length units) of aircraft a from airfield v to parcel p treated by way w;
- to each arc $(v,w) \in E$, where $v \in V(p,a)$ and $w \in L$, a positive real weight b_{vw} is associated as the length of flight (in length units) of aircraft a from parcel p treated by way v to airfield w;
- to each arc $(v,w) \in E$, where $v \in V(p_1,a)$ and $w \in V(p_2,a)$, a positive real weight b_{vw} is associated as the length of flight of aircraft a from parcel p_1 treated by way v to parcel p_2 treated by way w.

Now the problem OTALA can be reduced to the following multi-depot vehicle routing problem on graph G: find a subset A' from set of aircraft A and family $F = \{C_a, a \in A' \subseteq A\}$ such that:

- C_a is a cycle in graph G containing only one airfield node from L;
- for each $a_1, a_2 \in A'$ cycles C_{a_1} and C_{a_2} do not have a common parcel node from $N \setminus L$;
- for each parcel $p \in P$ there is only one node from $\bigcup_{a \in A} V(p,a)$ which belongs to a cycle from family F;
- for each $a \in A'$ the total amount (in volume units) of the chemicals demanded by all parcel nodes from cycle C_a does not exceed the chemical reservoir capacity of aircraft a;
- for each $a \in A'$ the total length of cycle C_a (determined as a sum of weights of its nodes and arcs) demands the total fuel consumption (in volume units) which does not exceed the fuel reservoir capacity of aircraft a;
- the total cost of all cycles from family F (the total cost of airfield activation plus total cost of fuel consumption) is minimised.

Graph G, associated to the problem OTALA with one airfield, two parcels and two aircraft is partially represented in Figure 2. Nodes v_{ij} represent way of treatment i for parcel j by the corresponding aircraft. Each non oriented edge represents two oriented

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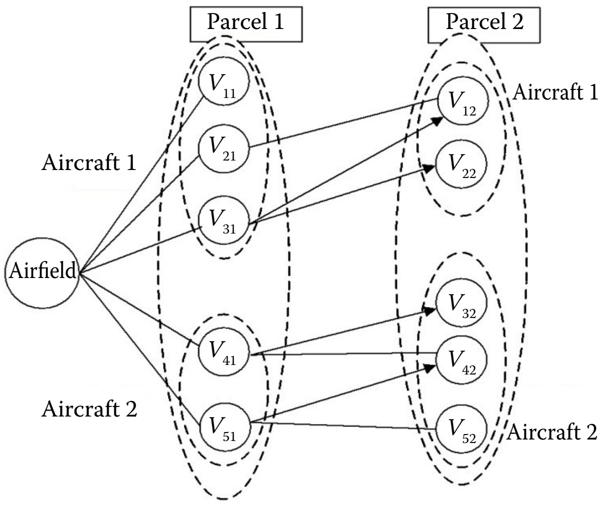

Figure 2. Graph representation of the problem (with one airfield)

nodes v_{ij} represent way of treatment i for parcel j by the corresponding aircraft

Source: authors' composition

arcs, while arcs between airfield node and nodes of parcel 2 are omitted.

From this graph it can be seen that node v_{11} is not connected by any node of parcel 2, i.e. for aircraft 1 there are no technically possible flights from parcel 1 treated by way 1 to parcel 2 for any way of its treatment. On the other hand, oriented arc (v_{41}, v_{32}) means that there is a technically possible flight of aircraft 2 from parcel 1 treated by way 4 to parcel 2 treated by way 3, but not *vice versa*. Non oriented edge $\{v_{41}, v_{42}\}$ means that there are technically possible flights of aircraft 2 between parcel 1 treated by way 4 and parcel 2 treated by way 4 in both directions.

Graph G can be defined in cooperation with experienced pilots/experts who determine technically possible flights for each two parcels. This is the best way how to reduce the maximum number of arcs of graph G and to stay in accordance with the real problem.

Mixed integer programming model of OTALA problem

Starting from the graph representation of problem OTALA, this problem can be formulated as a mixed integer programming (MIP) model. The following additional notations are introduced to formalise the MIP model:

- $c_l, l \in L$: fixed cost for activation of airfield l;
- s: fixed fuel costs per fuel unit;
- g_a, $a \in A$: fuel consumption per length unit for aircraft a;
- q_a, $a \in A$: fuel reservoir capacity for aircraft a;
- h_p, $p \in P$: chemical demand for parcel p;
- k_a, $a \in A$: chemical reservoir capacity for aircraft a.

Let us consider the following sets:

- set $VA(a), a \in A$, as a set of all nodes from $N \setminus L$ corresponding to treatment ways of parcels which are realised by aircraft a, i.e. $VA(a) = \bigcup_{p \in P} V(p,a)$;
- set $VP(p), p \in P$, as a set of all nodes from $N \setminus L$ corresponding to treatment ways of parcel p, i.e. $VP(p) = \bigcup_{a \in A} V(p,a)$.

MIP model uses the following binary variables:

- $y_l, l \in L$: 1 if and only if airfield node l belongs to a cycle from family F, i.e. node l is activated;
- $f_{la}, l \in L, a \in A$: 1 if and only if aircraft a belongs to subset $A^{'} \subseteq A$ and the corresponding cycle C_a from family F contains airfield node l;
- $x_{vw}, v, w \in N$: 1 if and only if arc (v,w) belongs to a cycle from family F.

MIP model uses continuous variables:

- $u_{ap}, a \in A, p \in P$: for $a \in A^{'}$ it represents the total chemical demand of nodes in cycle C_a, from airfield node to node corresponding to parcel p, if it exists. Otherwise, $u_{ap} = 0$.

Now the following MIP model of the problem can be defined:

$$(\min) \sum_{l \in L} c_l y_l + s \sum_{a \in A} g_a \left(\sum_{\substack{v \in VA(a) \cup L \\ w \in VA(a) \cup L}} \left(b_{vw} + d_w \right) x_{vw} \right) \tag{1}$$

subject to:

$$\sum_{v:(v,w) \in E} x_{vw} = \sum_{v:(w,v) \in E} x_{wv}, \text{ for each } w \in N \setminus L \tag{2}$$

$$\sum_{a \in A} f_{la} \leq |A| \, y_l, \text{ for each } l \in L \tag{3}$$

$$\sum_{l \in L} f_{la} \leq 1, \text{ for each } a \in A \tag{4}$$

$$\sum_{v \in VA(a)} x_{lv} \leq f_{la}, \text{ for each } l \in L \text{ and } a \in A \tag{5}$$

$$\sum_{v \in VA(a)} x_{vl} \leq f_{la}, \text{ for each } l \in L \text{ and } a \in A \tag{6}$$

$$\sum_{\substack{v:(v,w) \in E \\ w \in VP(p)}} x_{vw} = 1, \text{ for each } p \in P \tag{7}$$

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$$u_{ap} - u_{ar} + k_a \sum_{\substack{(v,w)\in E \\ v\in V(p,a) \\ w\in V(r,a)}} x_{vw} \le k_a - h_r, \text{ for } p,r \in P \text{ and } a \in A$$

$$\text{such that } V(p,a) \ne \varnothing, V(r,a) \ne \varnothing \tag{8}$$

$$\sum_{\substack{v\in VA(a)\cup L \\ w\in VA(a)\cup L}} \left(b_{vw} + d_w\right) x_{vw} \le {q_a}/{g_a}, \text{ for each } a \in A \tag{9}$$

$$x_{vw} \in \{0,1\},\ f_{la} \in \{0,1\},\ y_l \in \{0,1\},\ u_{ap} \ge 0 \tag{10}$$

The objective Function (1) represents total costs of all cycles considered as a sum of the total cost of airfield activation and the total cost of fuel consumption and it has to be minimised. Minimising the fuel consumption of vehicles can influence to direct reduction in the operating cost of their routing (Gaur et al. 2013). Constraints (2) ensure that for each node $w \in N \setminus L$ number of arcs which enter node w is equal to number of arcs which exit node w. Constraints (3) enable that, for each airfield node $l \in L$, if there is an aircraft from $A^{'}$ with the corresponding cycle containing node l then this node is activated. Constraints that, for each aircraft $a \in A$, if $a \in A^{'}$ then the corresponding cycle C_a contains only one airfield node, are modelled by (4). If an aircraft a belongs to $A^{'}$ and the corresponding cycle C_a contains an airfield node $l \in L$, then Constraints (5) ensure in C_a the existence of an arc which exit node l, while Constraints (6) ensure in C_a the existence of an arc which enter node l. Constraints (7) enable that, for each parcel $p \in P$ there is only one arc which belongs to a cycle from family F and enter a node corresponding to parcel p, i.e. a node from set $VP(p)$. Constraints (8) represent the well-known MTZ (Miller–Tucker–Zemlin) sub tour elimination constrains (Desrochers and Laporte 1991) adapted to our problem. The additional real variables u_i are used to give an ordering to all nodes excluding the depot to prevent the formation of illegal sub tours (Sawik 2016). Authors in (Kulkarni and Bhave 1985) adapted the original MTZ sub tour elimination constrains to the CVRP where it is assumed that the demand at each node is either less than or at the most equal to the capacity of each aircraft. We used these adapted MTZ constrains in our problem. Constraints (9) ensure that for each aircraft $a \in A^{'}$ the total weight of the corresponding cycle C_a, equal to its total length, does not exceed the maximal possible length of flight for aircraft determined as q_a/g_a.

Let us mention that if, for an $l \in L$ and all $a \in A$, $f_{la} = 0$, then this situation should imply $y_l = 0$, what is not modelled as a constrain in Model (1–10). Namely, according to (3), in this situation y_l can be 0 or 1. As the objective Function (1) is minimised, then in an optimal solution with such a situation y_l will be always equal to 0.

The MIP Model (1–10) may not have a feasible solution, due to its sufficient number of available aircraft with low chemical or/and fuel reservoir capacities, which can cause that Constrains (8) or/and (9) cannot be satisfied.

Preliminary numerical results

In order to investigate a power of defined MIP model, the data base is prepared, and the largest dimensions of the data are 245 parcels, four treatment ways for each parcel and seven potential locations for aircrafts. In order to solve the model, one of the non-commercial open-source linear programming solvers, called GLPK (GNU Linear Programming Kit) is applied. Performances of the computers we used are given in Table 1. Some numerical experiments with 11 OTALA instances and its characteristics of smaller dimensions from the data base have been performed and they are summarised in Table 2. The associated graphs in all instances contain the maximal number of technically possible arcs, except in instance 9 (only 60.2% of arcs).

It can be seen that the execution time required to find an optimal solution increases with dimensions of instances. Moreover, for instances 10 and 11 of larger dimensions, the solver does not succeed to find any optimal solution. Although instances 9 and 11 have the same characteristics (excepting the number of arcs), the solver succeeds to solve to the optimality only

Table 1. Performance of the computers used for experiments

Performance	Computer 1	Computer 2
Model	Toshiba Satellite L855	desktop computer
Processor	Intel (R) Core (TM)i3-3120m CPU 2.50 GHz	Intel (R) Core (TM) i5-3350P CPU 3.10 GHz
RAM	4.00 GB (3.89 useful)	8.00 GB

Source: authors

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Table 2. Problem instances and solver times

Problem instance #	Properties (number)					Solver execution time
	parcels	treatment ways	aircraft	airfields	binary decision variables	GLPK Integer Optimiser, v4.52
1	2	4	2	2	258	< 1 s
2	3	4	2	2	282	< 1 s
3	3	4	4	2	562	2.122 s
4	4	4	4	2	994	33.22 s
5	5	4	4	2	1 554	420 s
6	6	3	4	1	1 189	8 951.8 s
7	6	3	3	2	1 004	4 025.9 s
8	6	3	4	2	1 338	15 290.246 s
9	6	4	4	2	1 354	1 407.032 s
10	6	4	4	1	2 045	not solved, > 10 h, out of memory
11	6	4	4	2	2 242	not solved, > 19 h, out of memory

GLPK – GNU Linear Programming Kit

Source: authors' own calculations

instance 9, due to smaller number of arcs in the associated graph.

The preliminary numerical results show that for larger dimensions the solver does not guarantee to find an optimal solution of MIP model. As such difficulties could be expected in the case of real life instances, it is reasonable to develop a heuristic approach to solving OTALA problem.

Special heuristic for solving OTALA problem

Respecting characteristics and specificities of problem OTALA we develop here a special heuristic algorithm for its solving which represents a modification of the well-known Clark-Wright algorithm (Clark and Wright 1964).

In order to formalise the heuristics, we use the notations previously given, and introduce additional notations: all possible elementary routes – S; set P_l of all parcels assigned to l, $l \in L$; set A_l of all possible aircraft corresponding to parcels from P_l, $l \in L$; set $A' \subseteq A_l$ – treating set of parcels $P' \subseteq P_l$, $l \in L$; P_a – routes for aircraft a from set $A' \subseteq A_l$ treating set of parcels $P' \subseteq P_l$.

The pseudo-code of the heuristic:

Input (P, A, L, graph G)

(1) for sets of parcels P, aircraft A and airfields L form all possible elementary routes S and calculate their costs;

While $P \neq \emptyset$ and $A \neq \emptyset$ and $L \neq \emptyset$ do

(2) assign parcels from P to airfields from L;

(3) choose airfield $l \in L$ with the maximal number of assigned parcels;

(4) find set P_l of all parcels assigned to l and set A_l of all possible aircraft corresponding to parcels from P_l;

(5) find subset S_l of set S containing all elementary routes of the form $l - v - l$, where the parcel of node v belongs to P_l;

(6) for each possible pair of nodes of elementary routes from S_l calculate the "saving" amount. Sort all "savings" to a list according to non-increasing order;

(7) passing through the whole list of savings generate routes P_a for aircraft a from set $A' \subseteq A_l$ treating set of parcels $P' \subseteq P_l$;

(8) update sets P, A, L and S: $P = P \setminus P'$, $A = A \setminus A'$, $L = L \setminus \{l\}$, $S = S \setminus S_l$.

End

Output set of routes $\{P_a\}$.

Here we will give more details of some steps of the pseudo code.

In step 1 for each airfield $l \in L$, each parcel $p \in P$, each aircraft $a \in A$, where $V(p,a) \neq 0$, and each $v \in V(p,a)$, the algorithm forms the so called elementary route as the cycle $l - v - l$. This route is possible only if the chemicals demanded by parcel node v does not exceed the chemical reservoir capacity of aircraft a, and if the total length of the route demands the fuel amount which does not exceed the furl reservoir capacity of this aircraft.

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More formally, in order to be possible route $l-v-l$ should satisfy: $(b_{lv}+b_{vl}+d_v)g_a \le q_a$ and $h_p \le k_a$.

The cost of elementary route $l-v-l$ is equal to $s(b_{lv}+b_{vl}+d_v)g_a$.

Step 2 assigns each parcel $p \in P$ to an airfield $l \in L$ in the following way: for each airfield $l \in L$ and each aircraft for which $V(p,a) \neq 0$, consider costs of all elementary routes $l-v-l$ for $v \in V(p,a)$, and choose among them the minimal cost. Then, among all in such a way chosen minimal costs find the maximal value $m(l, p)$ over all aircraft. Now, parcel p is assigned to airfield $l^* \in L$ for which value $m(l, p)$ is minimised.

More precisely, in step 6 for each pair of nodes (p, q) which satisfy conditions:

- elementary routes $l-p-l$ and $l-q-l$ belong to set S_l;
- parcels corresponding to nodes p and q are different, but the corresponding aircraft is the same;
- there exists arc (p, q) in the graph associated to problem OTALA, the "saving" amount is calculated as $b+c-d$, where b is the cost of the flight from p to the current airfield l, c is the cost of the flight from l to q, and d is the cost of the flight from p to q.

If $b+c-d>0$, then pair (p, q) enters a list together with the corresponding "saving" amount.

In step 7 the algorithm passed through the whole sorted list of savings treating the current pair of nodes from the list in the following way: if (p, q) is the currently considered pair of nodes from the list, with aircraft a as the corresponding aircraft, then two cases are possible:

Case 1: If a route for aircraft a has not been formed yet, then investigate whether this aircraft has the sufficient fuel and chemical reservoir capacities to realise the route obtained by merging elementary routes $l-p-l$ and $l-q-l$. If it has, merge these elementary routes to the route which now represents the initial route P_a for aircraft a. Then, eliminate from the list each following pair of nodes such that their aircraft is different then a and at least one node from the pair has the same parcel as node p or node q.

Case 2: Let there exist a previously formed current route P_a for aircraft a, and let pair (p, q) be in the form (v, first) or (last, v), where there is no node in P_a with the same parcel as node v, *first* is the first node after node l and *last* is the last node before node l in route P_a. Investigate whether aircraft a can realise (with respect to fuel and chemical reservoir capacities) the route obtained by merging elementary route $l-v-l$ with route P_a (via node *first* or node *last*). If it can, P_a and $l-v-l$ are merged to the route which now represents a new route P_a. Then, as in Case 1, eliminate from the list each following pair of nodes such that their aircraft is nor aircraft a and at least one node from the pair has the same parcel as node p or node q.

After passing through the whole list of savings generated routes P_a represent the final routes for aircraft a from a subset of aircraft $A' \subseteq A$, which treat a subset of parcels $P' \subseteq P$.

The algorithm steps when at least one of updated sets P, A and L is empty, and the output is the family $\{P_a\}$ of all final routes generated during the algorithm's execution.

When set P of non-treated parcels is empty, the family $\{P_a\}$ represents a feasible solution of problem OTALA. If set A of all unused aircraft is empty, or the corresponding routes are formed for all airfields (i.e. $L=\varnothing$), it can happen that the family $\{P_a\}$ does not cover all parcels.

Numerical experiments with the heuristic

In order to investigate the efficiency of the proposed special heuristic algorithm, we applied this algorithm to 14 OTALA instances from our data base and the numerical results are presented in Table 3. We used JetBrains Platform (JetBrains 2018) (this platform is used in computer programming specifically for the Python language) and coded in Python language (version 3.5). Computer 2 (performances given in Table 1) is used for heuristic experiments. The characteristics of the instances are displayed in the first five columns of this table. In experiments we used 20 aircraft with capacity of 250, 10 with capacity of 200 and 10 with capacity of 150. Also, for each instance GLPK Integer Optimiser was applied to the corresponding MIP model and it found optimal solution always when it was possible.

In Table 3, columns 6 and 7 contain the execution times for GLPK, in the case it succeeded to find the optimal solution, and for the heuristic algorithm. It can be seen that GLPK solved only the first five instances within exponentially increased execution time, while the heuristic solved all instances within no more than 30 minutes for large dimensional problems, such as instances 11–14.

Columns 8–9 contain the optimal objective function value (if it was found) and the approximate objective function value obtained by the heuristic. We can measure the quality of the heuristic's solution using the following formula:

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Table 3. Execution time – solver *versus* heuristics

Problem #	Problem properties (number)				Execution time		Objective function value		
	parcels	ways of treatment	aircrafts	airfields	GLPK Integer Optimiser, v4.52	special heuristics	GLPK Integer Optimiser, v4.52	special heuristics	distance from optimal solution (%)
1	2	4	4	1	< 1 s	< 1 s	312.1	312.1	0
2	3	4	4	1	1 s	< 1 s	389.7	389.7	0
3	4	4	4	1	33 s	< 1 s	497.0	502.0	1.01
4	5	4	4	1	1 300 s	< 1 s	642.7	648.1	0.84
5	6	3	3	2	9 100 s	< 1 s	849.2	1 011.5	19.11
6	7	3	4	1	not solved (out of memory)	< 1 s	/	1 366.3	not applicable
7	10	4	4	1	not solved (out of memory)	< 1 s	/	1 324.0	not applicable
8	50	4	10	2	not solved	< 5 s	/	2 146.5	not applicable
9	100	4	10	2	not solved	40 s	/	8 016.07	not applicable
10	100	4	20	2	not solved	80 s	/	17 080.8	not applicable
11	150	4	35	2	not solved	15 min	/	25 939.8	not applicable
12	190	4	30	1	not solved	23 min	/	33 827.9	not applicable
13	198	4	40	1	not solved	25 min	/	37 416.7	not applicable
14	245	4	40	3	not solved	30 min	/	41 095.0	not applicable

GLPK – GNU Linear Programming Kit; symbol / indicates that the objective function value is not known because the problem is not solved

Source: authors' own calculations

https://doi.org/10.17221/134/2019-AGRICECON

$$\frac{f_h - f_{opt}}{f_{opt}} \times 100 \qquad (11)$$

where: f_h – the objective function value obtained by heuristic, and f_{opt} – the optimal objective function value. The last column in Table 3 contains the distances from the optimal solution, defined by (11), for instances 1–5. For the first two instances the heuristic reaches the optimal solution, while in other cases the distance is smaller than 20%.

The heuristic can obtain a solution which is not feasible, where some parcels are untreated. This situation can be overcome in reality, by increasing the number of aircraft or number of airfields.

CONCLUSION

This paper examines a specific problem of performing one operation on a parcelled agricultural land, which should be treated with the use of aviation. The studied problem consists of determining the optimal routes and it belongs to the class of NP-hard combinatorial optimisation problems and represents a variation of multi-depot vehicle routing problem.

The main results of the study are:

– new approach to the problem of optimal treatment of agricultural land using aviation is presented;
– the problem is explained in detail and presented on a graph;
– MIP model – a specific variant of multi-depot vehicle routing problem is formulated;
– although the exact solutions can be obtained only for smaller dimensions of the problem, the heuristic approach proved to be the only reasonable way to solve the problem instances of larger dimensions (the distance from optimal solution no larger than 20%);

– the numerical results have shown that the proposed model and approach can support farmers in selecting and performing land treatments.

The described approach to modelling and solving of a complex agricultural problem, considered in this paper, can provide a basis for some possible applications to other problems of the similar nature, as fire extinction in forestry and mosquito spraying. Unmanned aerial vehicles (UAV) are very popular for various applications in precision agriculture, so the modelling of the presented multi-depot vehicle routing problem, with some modifications, could be applied for solving UAV problems.

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Received May 15, 2019
Accepted August 28, 2019