A tool applicable to the payment of credits for projects of agricultural crops with different income levels

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Abstract: Traditionally, the banking sector has not accounted for the temporary loss in a customer's income, at any stage of the credit life, caused by the changes or the loss of employment, income drops in business, the establishment and development of new projects of agricultural crops with different income levels, or other contingencies that can arise in the today's economy. To address this problem, the present study constructs a phased model of one mother equation, from which a series of equations of financial mathematics are derived as a response to several needs of credit beneficiaries. The proposed model consists of one scenario, based on a mother equation. The scenario corresponds to credits with reduced or increasing payment instalments, postponable payment periods. Of the mother equation, 8 explicative variables were solved for a total of 9 phasing formulas for credits with three levels of payment. Our model, in contrast to the traditional one, incorporates postponable payment periods and jumps in payment instalments in any period of the lifetime of the credit due to a temporary loss in the customer's income and changes in the credit user's income.

Key words: welfare economy, agricultural crops with different income levels, jump discontinuity, banking

This study aimed at constructing a financial model that would allow banks to take into consideration the temporary difficulties that credit users face by introducing, at any time, periods of non-payment and payment in greater or lesser instalments according to the customer's income. The model aims to strengthen the bank-customer relationship, which would have the following effects, among others: increased lending by banks, increased investment in the economy, and increased savings. For the latter, Keynes asserted that all savings (S) become investments (I) (Keynes 1951, 1996; Costabile 2009; Pech and Milan 2009). Keynes also explained that the gap between S and I causes economic cycles and crises (Keynes 1951, 1996). In this context, Kaldor (1996) stated that the gap between S and I can be measured in the terms of growth and that welfare is not possible without economic growth. Therefore, welfare is a function of growth and other variables such as food, health, education, and political infrastructure. Thus, the greatest economic impact of the system of equations proposed in this study is a significant contribution to the welfare of the society because S and I are related in the market between savers (supply) and investors (demand). Furthermore, they are macroeconomic variables that increase a country's production capacity (Chong 2009), as Kaldor also argued in his work on economic efficiency (Kaldor 1961, 1980, 1996).

The fundamental mathematical input of this study's model is compound interest, which was officially introduced by Richard Witt in 1613. In 1671, the mathematician Johan de Witt returned to the topic in his work, 'The Value of Lifetime Revenue compared with Rescue Bonds'. This work was complemented by the great mathematician and astronomer Edmund Halley, a friend of Isaac Newton, in his study of life annuities. Since then, there is no evidence of a further research on this topic by the banking industry or financial economists.

This study's model, in contrast to the deferred uniform series, incorporates, at any moment of the credit lifetime, jumps in payment instalments due to the changes in the credit user's income and inserting payment discontinuities. These discontinuities, when the customer does not make any payment, are termed 'postponable payment periods' (*ppp*). These discontinuities solve the temporary periods when the credit user loses income due to circumstances such as the job relocation, non-reimbursed franchises, the loss of agricultural crops because of natural disasters, or other factors. The logic behind the modelling equations in the present study rests on a principle that allows the Paretian improvements; that is, when there are changes in the income of credit's user and payment discontinuities, their payment instalments must also change and to incorporate the postponable payment periods (Ramírez-Ceballos and Valencia-DeLara 2011a, b). In this way, bank customers or borrowers, when facing a decrease in income, do not sacrifice their consumption levels by having to pay constant instalments to the financial system as a whole.

In the present study, three uniform series are shown for one credit and the terms 'Phasing' and 'Payment Discontinuities' are coined in homage to the discontinuous functions of the mathematicians Jean-Baptiste-Joseph Fourier, Josiah Willard Gibbs, Peter Gustav Lejeune Dirichlet, who introduced the terms when defining a function $f(x)$ for all x except for x_0 and when finding the left and right limit of x_0, they found that one of the three possibilities is the 'jump' discontinuity (Bloomfield 1976; Coblentz 1988; Howell 2001; Gibbs 2008; Dirichlet 1997, 2010; Dufete 2010). Albert A. Michelson, Nobel Prize in Physics (1907), observed also the 'jump' type function near the discontinuity point, when his harmonic analyzer determined the 80 first components of the Fourier series.

These approaches, coming from pure mathematics, provide a strong argument to assert that a series of uniform payments does not necessarily have to be continuous throughout the entire lifetime of the credit, since the credit user can have a change in income and thus a finite payment discontinuity must be inserted (Gibbs' phenomenon) along with jumps in the payment instalment (Gibbs 2008). Thus the name of our study: Credits with three levels phasing, with postponable payment periods.

It is logical to think that the payment instalments of the credit beneficiary will change when his or her income changes and inserts *ppp*. The instalment differences cause jumps, which are called the Phasing of Uniform Series in this study, and the *ppp* is a discontinuity in payments. A first jump in the payment triggers a second phase and a second jump produces a third phase, which is separated by a discontinuity in payments. A discontinuity in payments is denoted as *ppp*, and it happens when there exists a temporary loss in the customer's income, and a jump in the payments is directly related to the debtor's improved future income (Phased Growth –*PG*–) or a decreased future income (Decreasing Phasing –*DP*–) (Ramírez-Ceballos and Valencia-DeLara 2011a, b).

The practical applications of the phasing system proposed in this study for credits with three levels of payments can be summarised as follows: the system responds to the real needs of the credit beneficiary by allowing the insertion of *ppp* for cycles in which the user temporarily does not have income, due to the circumstances (e.g., the loss or change of employment, a family tragedy, or the establishment and development of new projects of crops with different income levels) that can occur in the today's society. The model allows the credit user, in agreement with the bank, to plan his or her payments according to his/her current or future needs. Similarly, this model removes the complicated processes of legal actions that the banks initiate against borrowers who are temporarily behind in their payments for reasons beyond their control.

This study's model is also useful for investment alternatives, in which the projects go through the periods of economic disequilibrium because the benefits are not received (e.g., in the establishment of new agricultural projects with different income levels, as the cultivation of natural rubber, palm oil, coffee or olive, etc.) (Metin and Akin 2010). In these circumstances, when no economic benefits can be gained, the creation of the postponable payment periods is also beneficial.

The implementation of the model does not affect the bank's profitability, and the postponement of some payments does not financially affect the borrower because they are uniformly distributed in future periods. These postponements are based on the principles of the time value of money (*TVM*), financial equality, and the bank's opportunity interest rate (*OIR*).

This study considered, to better meet the different needs of the credit beneficiary, the following scenario: Credits with three levels of payments with increasing in the payment instalments, inserting *ppp* and without changing the time.

The above scenario generated one main equation of the updated value for the three payment series, which is called the mother equation. We solved the explicative or independent variables for this mother equation and obtained a group of 8 equations. In addition, the study analysed a real-life case study.

CREDITS WITH THREE LEVELS OF PAYMENTS WITH INCREASING IN THE PAYMENT INSTALMENTS, INSERTING *ppp* AND WITHOUT CHANGING THE TIME

The elements of the phasing model in this study are based on formulas and concepts in the existing literature: compound interest, the principles of financial equivalence and the time value of money (*VMT*), the interest rate conversions, additive and distributive properties, empowerment, setting, logarithms, and the basic algebra operations, among others (Allen 1965;

Weber 1976; Bhachman 1992; Varian 1993; Abell and Braselton 1994; Mignon 1998; Wolfran 1999; Don 2000; Cowan et al. 2002; Burnecki et al. 2003; Basilly et al. 2006; Brealey et al. 2006; Bodiel et al. 2007; Dodge 2007; Esch 2008; Hansen and Imrohoroglu 2008; Neftci 2008; Hutin 2010; Gong and Webb 2010; Horneff et al. 2010).

Variables corresponding to the Phasing Model

The following interrelated variables explain the model. They include the dependent or explained variable for the mother equation and the independent or explicative variables for the equations for the loans of the three payment levels. They are defined as follows:

a_1 = Value of the first round of uniform payments
i = Periodic interest rate
a_2 = Value of the second round of uniform payments
a_3 = Value of the third round of uniform payments
n = Number of payments of the first annuity a_1
n_1 = Number of payments of the second annuity a_2
m_2 = Number of payments of the third annuity a_3
k_1 = Value of the first payment discontinuity ppp_1
k_2 = Value of the second payment discontinuity ppp_2
n = Number of periods remaining after the first n payments of a_1 and the first postponable payment period (k_1 or ppp_1). It is calculated with the equation $m_1 = T_1 - n - k_1$. We use this value to calculate a_2; m_1 is the number of payments of the second annuity a_2 if a_3 not occurs
m_2 = Number of periods remaining after subtracting from the number of payments of the second annuity m_1, the values of n_1 and k_2; that is, $m_2 = m_1 - n_1 - k_2$
PV = Updated value of the three uniform payment series (value of the loan)
N = The total number of payments in the credit life
M = The total number of length periods of the transaction in the credit with phasing
T_1 = The total number of periods in a credit before implementing phasing

Demonstration of the mother equation

In the following section, a timeline shows the movement of funds from the bank's point of view. The payments of the a_1, a_2 and a_3 annuities are the income for the bank and they are represented with arrows pointing up, and credit – which is a payment – is represented by an arrow pointing down. Annuity a_1 is updated by n periods; annuity a_2 is updated by $n + k_1 + n_1$ periods; and annuity a_3 is updated by $n + k_1 + n_1 + k_2 + m_2$ periods, as shown in Figure 1.

Figure 1 also shows the updates of annuities a_1, a_2 and a_3 from each future point to the zero origin of the timeline. These points are represented by arrows pointing to the left and are accompanied by the respective present value and compound interest formulas. The formulas in Figure 1 are shown according to the following convention and F is the Focal point:

$$A = \frac{a_1}{I^n};\ B = \frac{a_1}{I^{n-1}};\ C = \frac{a_1}{I^2};\ D = \frac{a_1}{I^1};\ E = \frac{a_2}{I^{n_1+k_1+n}};$$

$$F = \frac{a_2}{I^{n_1-1+k_1+n}};\ G = \frac{a_2}{I^{2+k_1+n}};\ H = \frac{a_2}{I^{1+k_1+n}};$$

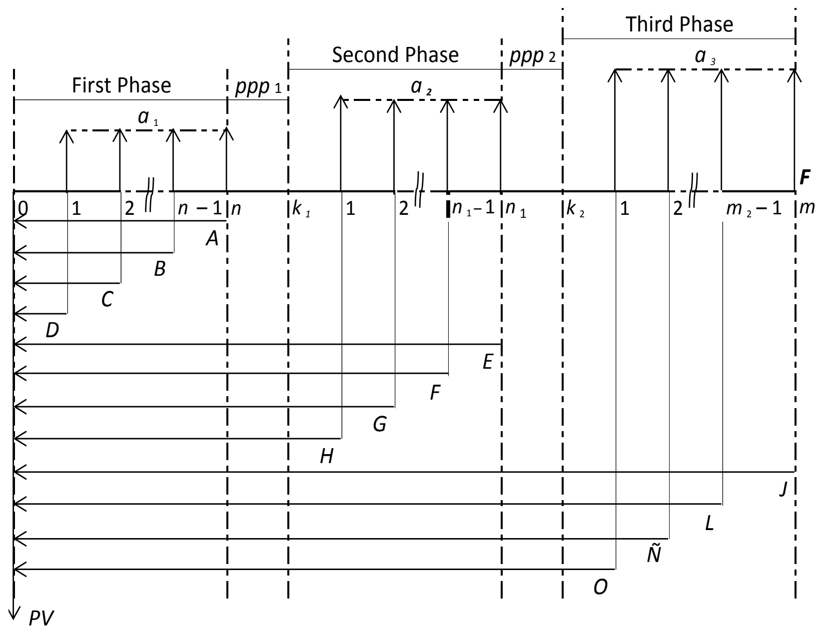


Figure 1. Present Value of three-phase uniform time series

$$J = \frac{a_3}{I^{m_2 + k_2 + n_1 + k_1 + n}}\,;\; L = \frac{a_3}{I^{m_2 - 1 + k_2 + n_1 + k_1 + n}}\,;$$

$$Ñ = \frac{a_3}{I^{2 + k_2 + n_1 + k_1 + n}}\,;\; O = \frac{a_3}{I^{1 + k_2 + n_1 + k_1 + n}}\,;$$

with $I = (1 + i)$

In what follows, we demonstrate the present value for the mother equation:

I. Updating in the zero period of the first annuity:

1. $PV_{a_1} = \frac{a_1}{I^n} + \frac{a_1}{I^{n-1}} + \cdots + \frac{a_1}{I^3} + \frac{a_1}{I^2} + \frac{a_1}{I^1}$

2. Multiplying 1 by I, yields the following equation:

$$PV_{a_1} \times I = \frac{a_1}{I^{n-1}} + \frac{a_1}{I^{n-2}} + \cdots + \frac{a_1}{I^2} + \frac{a_1}{I^1} + \frac{a_1}{I^0}$$

3. Point 2 is extracted from Point 1 and, when algebraically solved, yields the equation that updates the first annuity a at zero:

$$PV_{a_1} = \frac{a_1 \times [I^n - 1]}{i_p \times I^n} \qquad (1)$$

II. Updating the second uniform series of payments, a_2:

1. $PV_{a_2} = \frac{a_2}{I^{n_1 + k_1 + n}} + \frac{a_2}{I^{n_1 - 1 + k_1 + n}} + \cdots + \frac{a_2}{I^{2 + k_1 + n}} + \frac{a_2}{I^{1 + k_1 + n}}$

2. The incremental factor I, when multiplied by II.1, can be expressed as:

$$PV_{a_2} \times I = \frac{a_2}{I^{n_1 - 1 + k_1 + n}} + \frac{a_2}{I^{n_1 - 2 + k_1 + n}} + \cdots + \frac{a_2}{I^{1 + k_1 + n}} + \frac{a_2}{I^{0 + k_1 + n}}$$

3. Point 2 is extracted from Point 1, yielding the following mathematical expression:

$$PV_{a_2} - PV_{a_2} \times I = \frac{a_2}{I^{n_1 + k_1 + n}} - \frac{a_2}{I^{0 + k_1 + n}}$$

4. Solving by fraction subtraction, factoring, and other algebraic operations, we obtain the following equation:

$$PV_{a_2} = \frac{a_2 \times \left[(1 + i_p)^{n_1} - 1\right]}{i_p \times I^{n_1 + k_1 + n}} \qquad (2)$$

III. Updating at the zero point of annuity a_3:

1. $PV_{a_3} = \frac{a_3}{I^{m_2 + k_2 + n_1 + k_1 + n}} + \frac{a_3}{I^{m_2 - 1 + k_2 + n_1 + k_1 + n}} + \cdots + \frac{a_3}{I^{2 + k_2 + n_1 + k_1 + n}} + \frac{a_3}{I^{1 + k_2 + n_1 + k_1 + n}}$

2. Multiplying the binomial factor I by Point 1 yields:

$$PV_{a_3} \times I = \frac{a_3}{I^{m_2 - 1 + k_2 + n_1 + k_1 + n}} + \frac{a_3}{I^{m_2 - 2 + k_2 + n_1 + k_1 + n}} + \cdots + \frac{a_3}{I^{1 + k_2 + n_1 + k_1 + n}} + \frac{a_3}{I^{0 + k_2 + n_1 + k_1 + n}}$$

3. Point 2 is subtracted from Point 1 to obtain the expression in the following form:

$$PV_{a_3} - PV_{a_3} \times I = \frac{a_3}{I^{m_2 + k_2 + n_1 + k_1 + n}} - \frac{a_3}{I^{0 + k_2 + n_1 + k_1 + n}}$$

4. Solving with fraction subtraction, factoring, and other algebraic operations, we obtain the following equation:

$$PV_{a_3} = \frac{a_3 \times [I^{m_2} - 1]}{i_p \times I^{m_2 + k_2 + n_1 + k_1 + n}} \qquad (3)$$

IV. Subsequently, the updates of the three annuities a_1, a_2 and a_3 are added.

1. The sum of (1) + (2) + (3) is the following:

$$\sum_{i=a_1}^{a_3} PV_i = PV_{a_1} + PV_{a_2} + PV_{a_3}$$

2. Adding the former fractions, we obtain the following expression:

$$PV = \frac{I^{m_2 + k_2} \times \{I^{\,n_1 + k_1} \times a_1 \times [I^n - 1] + a_2 \times [I^{n_1} - 1]\} + a_3 \times [I^{m_2} - 1]}{i_p \times I^{m_2 + k_2 + n_1 + k_1 + n}} \qquad (4)$$

where Equation (4), in loans with three uniform payment phases, is used to update the three payment series a_1, a_2 and a_3, at zero. The solution is the amount of the loan. Because the explicative variables have been defined above, we will not dwell on this aspect. It should also be noted that, with the phased growth (*PG*), a_1 is paid n times, a_2 is paid n_1 times, and a_1 is paid m_2 times. The latter is noted as the number of payments of a_2 is $m_1 = T_1 - n - k_1$ (if c not occurs). The third annuity is paid m_2 times and it is noted as the number of payments of $a_3 = m_2$.

The following section shows the interpretation of the exponents of the $I = (1 + i_p)$ binomials in equation (4):

1. The exponent $m_2 + k_2 + n_1 + k_1 + n$ of the factor I in the denominator reflects the time that the customer takes to pay his or her credit with phasing. It is represented by the letter M and leads to the following equation: $M = m_2 + k_2 + n_1 + k_1 + n$.
2. The exponents n, n_1 and m_2 of the 3rd, 4th, and 5th I factors indicate the number of payments of annuities a_1, a_2 and a_3, respectively. Therefore, the total number of payments in a three-phase credit is $N = n + n_1 + m_2$.

In addition, for this scenario or the mother equation, the relationships between M, N, T, k_1 and k_2: are considered.

Table 1. Credit data

a_1 = €1084.13	$m_1 = T_1 - n - k_1$ = 31 months
a_2 = €1057.54	$E_{a2} = m_1 + t_1$ = 32 + 2 = 33 months. Number of payments of a_2 if a_3 not occurs
a_3 = €1121.38	k_2 = 1 month
n = 8 months	$m_2 = m_1 - n_1 - k_2$ = 31 – 10 –1 = 20 months
k_1 = 1 month	$E_{a3} = m_2 + t_2$ = 20 + 3 = 23 months. Number of payments of a_3 because this is the last annuity
T_1 = 40 months	$m_2 = m_1 - n_1 - k_2 + n$ = 31 – 10 –1 = 20 months
n_1 = 10 months	$N = n + n_1 + m_2$ = 8 + 10 + 20 = 38 payments
	$M - N = k_1 + k_2$ (the equation is satisfied)

1. $M = m_2 + k_2 + n_1 + k_1 + n$, which means that the credit term (*WP – With Phasing*) has N periods of payments and $k_1 + k_2$ periods of no payments;
2. $T_1 = N + k_1 + k_2$, which means that the number of payments for the loan with Phasing is reduced by $k_1 + k_2$ periods.
3. 1 and 2 yield the equation $m_2 + k_2 + n_1 + k_1 + n = N + k_1 + k_2$; then $N + k_1 + k_2 = N + k_1 + k_2$ because $N = m_2 + n_1 + n$.
4. 3 concludes that $M = T_1$, which proves that the deadlines with and without phasing are equal.

It should be clarified that m_1 does not appear in (4) because the n_1 payments of a_2 are interrupted in the n_1 period before the second ppp_2. After ppp_2, m_2 payments of a_3 are made.

Equations (5) to (12), shown in the Appendix, correspond to the solutions of the independent variables for equation (4).

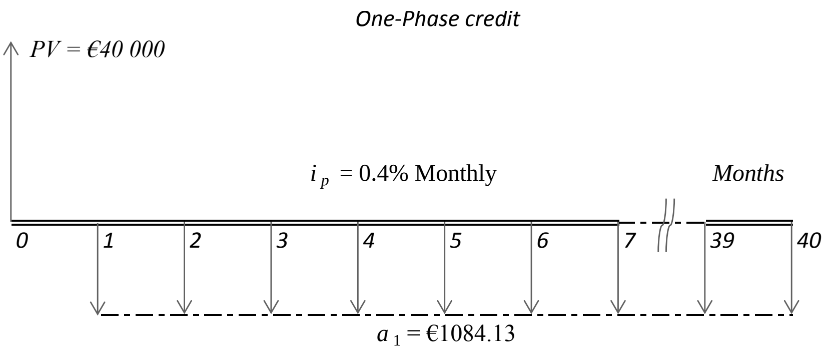
Application example for equation (4)

Initially, a bank and a customer enter a creditor-debtor relationship for a 40-month period and the bank charges an interest of 4.8% (0.4% monthly). The credit must be paid in 40 uniform periodic instalments. Later, the credit beneficiary reaches a new agreement with the bank to make credit payments according to the following plan: 8 monthly uniform instalments of €1084.13 each, followed by a postponable payment period of 1 month; 10 payments of €1121.38 each, followed by 1 month without any payments; and, during the last 20 months, instalments of €1179.83 each. Solution in this problem (Table 1 shows the problem data):

(a) Solution of the equation $m_1 = T_1 - n - k_1$ (# of pending payments);
(b) Value of the loan (keeping in mind that the credit has three phases and two jumps in payments);
(c) Figures of the Flow of Revenue without and with Phasing;
(d) Amortisation table without and with jumps in payments (without and with Phasing).

Solution:

(a) Calculation of m_1: $m_1 = T_1 - n - k_1$ = 40 – 8 – 1 = 31, which is the number of periods remaining after the first payment cycle and the ppp_1.
(b) Calculation of the credit value: Substitute the explicative variables in (1), PV = €40 000.00. This process yields the updated value (in month 0) of the three annuities, which means that it is the same or equivalent for the bank to receive €40 000 now, or to receive payments of a_1 = €1084.13 each for the first 8 months of loan; 10 payments of a_2 = €1121.38 each and 20 payments of a_2 = €1179.83 each, if the opportunity interest rate is 0.4% monthly, and two non-payment periods of one month each. In this case the inequality $a_1 > a_2 > a_3$, being this credit with increasing phasing.



Figures 2. Flow of revenue without phasing – from the customer's point of view

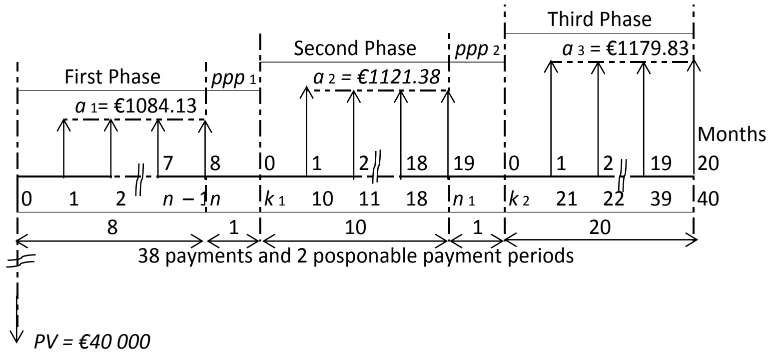


Figure 3. Flow of revenue with increasing phasing

Figure 2 shows a one-phase credit term and it indicates that the credit user or beneficiary pays equal instalments of €1084.13 during the entire credit lifetime. Thus, no phasing is necessary. A three-phase loan is shown in Figure 3. This figure indicates that the first phase is made up of 8 payments of a_1 = €1084.13 each, followed by the first ppp_1 of one month. The second phase is made up of 10 payments of a_2 = €1121.38 each, followed by the second ppp_2 of one month. The third and final phase is made up of the last 20 payments of a_3 = €1179.83 each.

(c) Credit Amortisation without and with jumps or phasing in payments:

Table 3 shows the behaviour of credit. This table shows the first 8 payments of annuity a_1 with a value of €1084.13 each; ppp_1 of 1 month; 10 payments of the second annuity a_2 with a value of €1121.38 each; ppp_2 of 1 month; and the last 20 payments of annuity a_3 with a value of €1179.83 each. This credit decreases in $k_1 + k_2 = 2$ *Months* the number of payments that were agreed upon before phasing (from 40 to 38 payments in our example). For the example given, the number of payments is as follows: 8 payments of a_1, 10 payments of a_2, and 20 payments of a_3, with 2 months of the postponable payment periods.

CONCLUSIONS

This study achieved one of its main goals, which was to construct a financial model that would allow banks to adjust to the temporary difficulties that affect credit users by introducing non-payment periods and greater or lesser instalments according to the customer's income. This model can be applied to new projects of agricultural crops with different income levels such as the natural rubber cultivation, olive, coffee or palm oil, etc. The proposed model consists of one scenario, based on a mother equation. The scenario corresponds to credits with reduced or increasing payment instalments and postponable payment periods. Of the mother equation, 8 explicative variables were solved for a total of 9 phasing formulas for credits with three levels of payment (Equations (4) to (12)).

The group of 9 equations in the Phased System of Uniform Time Series with three levels of payment, shown in the Appendix, represents different alternatives that the banks have to resolve various situations with their clients. The model responds to the changes in the income and a temporary loss in the customer's income and thus improves the bank-customer relationship. It increases credit and investment and generates collective welfare in the society by causing a natural optimisation process (Buyukkarabacak and Valev 2010;

Table 2. Credit amortisation without jumps or phasing in payments in €

Credit Amount:	€40 000.00	Interest rate	0.40%	Monthly
Term of loan:	40	T. WOP	60	Months
Periods	Interest	Capital	Instalment	Balance
0				40 000.00
1	160.00	924.13	1 084.13	39 075.87
10	126.19	957.93	1 084.13	30 590.60
20	87.18	996.95	1 084.13	20 797.98
30	46.58	1 037.55	1 084.13	10 606.53
39	8.62	1 075.51	1 084.13	1 079.81
40	4.32	1 079.81	1 084.13	0.00

Table 3. Credit amortisation with three payment phases in €

Credit Amount:	€40 000.00	Interest rate	0.40%	Monthly
Term of loan:	40	T. WP:	40	Months
n	8	k_1	1	Month
m_1	31	k_2	1	Month
m_2	20	n_1		
Periods	Interest	Capital	Instalment	Balance
0				40 000.00
1	160.00	924.13	1 084.13	39 075.87
5	145.13	939.00	1 084.13	35 342.25
8	133.81	950.31	1 084.13	32 502.65
9		First postponable payment periods (k_1)		32 632.66
10	130.53	990.85	1 121.38	31 641.81
11	126.57	994.81	1 121.38	30 647.00
15	110.55	1 010.83	1 121.38	26 627.79
19	94.28	1 027.10	1 121.38	22 543.88
20		First postponable payment periods (k_2)		22 634.06
21	90.54	1 089.30	1 179.84	21 544.76
22	86.18	1 093.66	1 179.84	20 451.10
30	50.69	1 129.15	1 179.84	11 542.89
35	27.92	1 151.91	1 179.84	5 829.04
39	9.38	1 170.45	1 179.84	1 175.13
40	4.70	1 175.13	1 179.84	0.00

Gutierrez and Kales 2010). The implementation of this model benefits elements of the society without harming others, which is consistent with the notion of the Pareto efficiency (Pareto 1980; Domínguez 2010).

Our model, in contrast to the traditional one, incorporates postponable payment periods and jumps in payment instalments in any period of the lifetime of the credit due to a temporary loss in the customer's income and changes in the credit user's income. Thus, it is a dynamic system that can respond to changes in the income of the credit beneficiary and to a temporary loss in the customer's income.

This study model could be of great use in both developed and developing economies. In economies with a strong occupational mobility of workers, job seekers often face periods without income. Their solution is to incorporate postponable payment periods into their payment programmes with banking institutions. Additionally, the proposed system could help to create and consolidate agricultural projects. The proposed system offers an opportunity for the banks (when granting credits with postponable payment periods for the creation and consolidation of agricultural crops projects) to play a central role in economic development and job creation, as long as the budding agricultural entrepreneur can create a payment plan to respond to the present and future needs of his or her project.

APPENDIX

Equations for credits with three phases:

I. Equations for the first scenario:

$$PV = \frac{I^{m_2+k_2} \times \{I^{n_1+k_1} \times a_1 \times [I^n - 1] + a_2 \times [I^{n_1} - 1]\} + a_3 \times [I^{m_2} - 1]}{i_p \times I^{m_2+k_2+n_1+k_1+n}} \tag{4}$$

$$a_1 = \frac{I^{m_2} \times \{PV \times i_p \times I^{k_2+n_1+k_1+n} - a_2 \times I^{k_2} \times [I^{n_1} - 1] - a_3\} - a_3}{I^{m_2+k_2+n_1+k_1} \times [I^n - 1]} \tag{5}$$

where the exponent $m_2 + k_2 + n_1 + k_1$ of I in the denominator of equation (5) is the number of periods that occur after n.

$$a_3 = \frac{I^{m_2+k_2} \times \{I^{n_1} \times (PV \times i_p \times I^{k_1+n} - I^{k_1} \times a_1 \times [I^n - 1] - a_2) + a_2\}}{[I^{m_2} - 1]} \tag{6}$$

where the exponent $m_2 + k_2$ of I in the denominator of equation (6) is the number of periods that occur after n_1.

$$a_3 = \frac{I^{m_2+k_2} \times \{I^{n_1} \times (PV \times i_p \times I^{k_1+n} - I^{k_1} \times a_1 \times [I^n - 1] - a_2) + a_2\}}{[I^{m_2} - 1]} \tag{7}$$

$$n = \frac{\log\left\{\dfrac{a_3 \times [I^{m_2} - 1] + I^{m_2+k_2} \times \{a_2 \times [I^{n_1} - 1] - a_1 \times I^{n_1+k_1}\}}{I^{m_2+k_2+n_1+k_1} \times \{PV \times i_p - a_1\}}\right\}}{\log I} \tag{8}$$

where the exponent $m_2 + k_2 + n_1 + k_1$ of I in the first denominator of equation (8) is the number of periods that occur after n.

$$n_1 = \frac{\log\left\{\dfrac{a_3 \times [I^{m_2} - 1] - a_2 \times I^{m_2+k_2}}{I^{m_2+k_2} \times \{PV \times i_p \times I^{k_1+n} - I^{k_1} \times a_1 \times [I^n - 1] - a_2\}}\right\}}{\log I} \tag{9}$$

$$m_2 = \frac{\log\left\{\dfrac{a_3}{I^{k_2+n_1+k_1} \times (a_1 \times [I^n - 1] - PV \times i_p \times I^n) + a_2 \times I^{k_2} \times [I^{n_1} - 1] + a_3}\right\}}{\log I} \tag{10}$$

$$k_1 = \frac{log\left\{\dfrac{a_3 \times [I^{m_2} - 1] + a_2 \times I^{m_2+k_2} \times [I^{n_1} - 1]}{I^{m_2+k_2+n_1} \times \{PV \times i_p \times I^n - a_1 \times [I^n - 1]\}}\right\}}{\log I} \tag{11}$$

$$k_2 = \frac{log\left\{\dfrac{a_3[I^{m_2} - 1]}{I^{m_2+n_1+k_1}.\{PV \times i_p \times I^n - a_1 \times [I^n - 1]\} - a_2 \times I^{m_2} \times [I^{n_1} - 1]}\right\}}{\log I} \tag{12}$$

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