Project costs planning in the conditions of uncertainty

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Abstract: The fuzzy approach to the project network analysis of the project planning and control is commonly oriented on the fuzzy critical path setting and the project duration monitoring. In the article, this approach is improved by the addition of the project costs perspective to the standard time aspect. The relations for the fuzzy quantity of the total project costs and for its membership function are derived. The example demonstrates the application of the theoretical relations and shows how the enhanced fuzzy approach can be used when different project variants are to be compared. The example also reveals how the fuzzy approach with the project costs monitoring brings new information for the project planning and management and for the risk management.

Key words: project planning and management, fuzzy methods, simulations of uncertainty, costs planning, project costs, project network analysis, critical path, risk management, support for decisions, project variants

Agricultural enterprises are, like other business subjects, facing many challenges and demands of the present severe economic situation. The knowledge of the effective management techniques may represent an important competitive advantage and essentially help to achieve the objectives of business strategies. Many research studies were focused on the investigation of the new technologies adaptation in the agri-food sector companies – cf. Nagy et al. (2009), Demiryürek (2010), Tomšík and Svoboda (2010) etc. The research proves that the modern information technologies and systems as well as good project and risk management techniques can considerably contribute to the competitiveness of enterprises specialising in agricultural production, the same as in other economic branches – for example Vrana (2006), Hennyeyová and Depeš (2010).

The techniques of the project network analysis belong to the core techniques used in project planning and control. They enable to describe and optimize the complex structure of project activities, to prepare the project time schedule, to plan project costs, to allocate resources to activities and then to control all activities to the successful completion of the specific project goals and objectives. We can divide these techniques into several groups according to their approach to the quantitative indicators describing the project activities, above all to the activity duration. The techniques from the first group are based on the deterministic setting of all activity durations. Typically, this group is represented by the techniques of the CPM (Critical Path Method) and MPM (Metra Potential Method). These techniques are suitable especially for more or less repetitious projects, when the activity durations are known, determinable, or well predictable. Nevertheless, every project is unique, the conditions for every project are unique, too, and the project progression may be influenced by many unexpected effects. In an effort to incorporate the accidentality into the activity description, the second group of the project network techniques was developed. The activity duration is described by a random value with the probability distribution on a particular interval around the mean value. These techniques are represented above all by the PERT (Program Evaluation and Review Technique). However, to be able to choose the correct probability model, it is necessary to have enough statistical data. Nevertheless, on principle, it is difficult to meet this condition for a unique project. For this reason, the third group of project network techniques based on the theory of fuzzy sets and fuzzy quantities may be more appropriate.

The idea to apply the fuzzy theory in the project description is not new, but the fuzzy approach was usually used to capture the uncertainty in project duration and critical path setting – e.g. Chanas and Zieliński (2001), Liberatore and Connelly (2001), Mareš (2002), Sireesha et al. (2010). These ideas are extended in the present article introducing the fuzzy approach also to the project costs description.

FUZZY SETS AND FUZZY QUANTITIES

To simplify understanding of the next constructions, at first we summarize some basic terms and relations of the fuzzy theory underlying the third group of the project network techniques and used in further explanation. A more complex description of the fuzzy sets and fuzzy quantities theory can be found in the works deeply focused on this topic, for example in Zadeh (1965), Wang and Kerre (2001), Novák (1990) and Mareš (2002).

The fuzzy set conception was introduced as an extension of the classical notion of the set. In the case of an ordinary deterministic set, we can definitely decide if an element belongs or does not belong to the set. Fuzzy sets enable – in addition to these two "sharp" possibilities – to express the state when an element belongs to a set only partially, to a certain degree. This definition of the set with vague boundaries enables us to describe many situations from the daily life by propositions like "more distant building", "light green" or "a little longer".

Another generalization introduced for fuzzy sets is the notion of membership function. It extends the notion of the characteristic function used for the classical sets. A fuzzy set A, the subset of universe $\mathcal{U}$, can be uniquely described by its membership function $\mu_A(x)\colon \mathcal{U} \to \langle 0;1\rangle$

$\mu_A(x) = 1$ when x definitely belongs to A
$\mu_A(x) = 0$ when x definitely does not belong to A
$0 < \mu_A(x) < 1$ when it is not definite whether x belongs to A or not

And the nearer the value of $\mu_A(x)$ is to unity, the higher is the grade of the membership of x in A.

The membership function enables to define the operations between fuzzy sets and notions representing the fuzzy extensions of operations and notions known for the ordered sets. Further, we will utilize the definition of the union and the intersection of two fuzzy sets A, B

union of fuzzy sets $A \cup B$

$$\mu_{A\cup B}(x) = \max\,(\mu_A(x), \mu_B(x)) \quad \text{for all x } \textit{from } \mathcal{U} \quad (1)$$

intersection of fuzzy sets $A \cap B$

$$\mu_{A\cap B}(x) = \min\,(\mu_A(x), \mu_B(x)) \quad \text{for all } x \text{ from } \mathcal{U} \quad (2)$$

Another generalization consists in the definition of fuzzy quantity as an extension of the notion of number. The fuzzy quantity a is a subset of real numbers $\mathcal{R}$ (or integer numbers $\mathcal{Z}$) with the membership function μ_a complying with the following condition: there is a number $x_a \in \mathcal{R}$ with $\mu_a(x_a) = 1$ and simultaneously there are numbers $x_a^{(1)}, x_a^{(2)} \in \mathcal{R}$, $x_a^{(1)} < x_a < x_a^{(2)}$ with $\mu_a(x) = 0$ for all $x < x_a^{(1)}$ and for all $x > x_a^{(2)}$. The number x_a is called the modal value of the fuzzy quantity a.

The fuzzy quantity $-a$ is called inverse to a fuzzy quantity a if its membership function satisfies the following condition:

$$\mu_{-a}(x) = \mu_a(-x) \quad \text{for all } x \in \mathcal{R} \quad (3)$$

The operation addition of fuzzy quantities may be defined as follows. The sum of two fuzzy quantities a and b is also a fuzzy quantity, we denote it by $a \oplus b$ and its membership function is

$$\mu_{a\oplus b}(z) = \sup\,[\min\,(\mu_a(x), \mu_b(y)) : x, y \in \mathcal{R}, \quad x + y = z] \quad \text{for all } z \in \mathcal{R} \quad (4)$$

In an analogical manner, we can define the fuzzy quantity as an extension of a real function of real variables. Let f be a real function of real variables $x_1, x_2, \ldots, x_n$. If we substitute real variables by fuzzy quantities, the function value of the generalized function is also a fuzzy quantity with the membership function

$$\mu_f(z) = \sup\,[\min\,(\mu_1(x_1), \mu_2(x_2), \ldots, \mu_n(x_n)] : z = = f\,(x_1, x_2, \ldots, x_n)] \quad (5)$$

Conditions for a linear space are satisfied for the operations defined in this way; see e.g. Mareš (2002). Next, we use the commutativity and associativity of fuzzy quantities addition.

To be able to operate with fuzzy quantities, we need to introduce the order relation, too. This problem can be treated in several ways – e.g. in Wang and Kerre (2001). Next, we use the order definition that can be found e.g. in Novák (1990) and Mareš (2002). We denote by $\{a, b\}_{a\geq b}$ the set corresponding to order relation of two fuzzy quantities a, b. Each of the fuzzy quantities a, b has generally more possible values with different grades of membership. Then the validity of the inequality has also different grades of membership and the set $\{a, b\}_{a\geq b}$ is a fuzzy set. This relation can be described verbally by the phrase "a is fuzzy greater than b" and be denoted by $a \circeq b$. Let us define the membership function $\mu_{a\geq b}$ of the fuzzy set $\{a, b\}_{a\geq b}$ as follows:

$$\mu_{a\geq b}(a, b) = \sup\,[\min\,(\mu_a(x), \mu_b(y)) : x, y \in \mathcal{R}, x \geq y] \quad (6)$$

It is obvious that $a \circeq b$ may hold with some possibility as well as $b \circeq a$ may also be valid with some possibility, i.e. that both expressions $\mu_{a\geq b}(a, b) > 0$ and $\mu_{b\geq a}(a, b) > 0$ may be simultaneously valid.

We have extended the classical quantity definition by the aspects of uncertainty. On the other hand, the enhancement and the order relation defined above have a certain drawback. For the general fuzzy quantities a, b, c, the order relation is no longer transitive. This problem is a topic for a detailed study in the fuzzy quantity theory and is not discussed here anymore. We refer the interested reader to the literature specialized in the topic of fuzzy quantities and the relations between them – see e.g. Dubois, Prade (2000), Hanss (2005).

CRITICAL PATH IN THE FRAMEWORK OF UNCERTAINTY

The third group of project network techniques mentioned above is based on the results of the fuzzy sets and fuzzy quantities theory. However, regardless whether we use these techniques or the techniques of other two groups, we should first have the project divided into particular activities which enables and simplifies the project planning and control. Some activities of the project may run concurrently, independently. Or some activities may be arranged in a sequence, which means that one activity cannot start before another immediately foregoing activity is (or before other immediately foregoing activities are) accomplished. This structure may be smartly described using the mathematical graph theory.

Suppose the project with $K + 1$ states, $\{0, 1, \ldots, K\}$, and n activities to be accomplished. Every activity means a transition from one state to another. Without the loss of generality it is possible to assume that there is just one state in the project (and also in the corresponding graph) that has no foregoing state, i.e. no activity ends in this state. Let us denote this first, starting state by 0. Similarly, it is possible to assume that there is just one state that has no following state. Let us denote this last, ending state by K. Further, let us denote by B_r the set of states immediately foregoing the state r

$$B_r = \{v_0, v_1, \ldots, v_m\} \quad \text{where } 1 \le m \le K$$

$$v_0, v_1, \ldots, v_m \in \{0, 1, 2, , \ldots, K\} \text{ and } r = 0, 1, 2, \ldots, K$$

From the mathematical point of view, we have defined a directed connected graph with vertices $0, 1, \ldots, K$ and directed edges $A_{vr} = (v, r)$, where $v \in B_r$.

Further, we introduce the notion of path. It is the sequence of the consecutive activities from the starting state to the end state, i.e. the sequence

$$P_s = \{A_{v_0v_1}, A_{v_1v_2}, \ldots, A_{v_{k-1}v_k}\}$$
$$\text{where } v_0 = 0,\ v_k = K,\ v_i \in B_{i+1} \quad \text{for all } i = 0, 1, \ldots, k-1$$

It follows from the graph definition that there is always at least one path like this one in this kind of graph. Nevertheless, the description of the real project structure contains usually many such paths.

Let us denote the graph edges representing the particular activities of project more simply by A_i, $i = 1, 2, \ldots, n$. Let t_i be the duration of the activity A_i, $i = 1, 2, \ldots, n$. Then the duration T_s of the path P_s of sequential activities is given by the sum

$$T_s = t_{i_1} + t_{i_2} + \ldots + t_{i_k}$$
$$\text{where } i_1, i_2, \ldots, i_k \in \{1, 2, \ldots, n\}, \quad 1 \le k \le n$$

To realize the whole project, it is necessary to accomplish all its activities A_i regardless in which path the activity is situated. However, it is useful for the project control to introduce the notion of the critical path as the path with the longest duration. It follows from the underlying idea that any prolongation of the duration of any activity lying in the critical path brings the prolongation of the whole project. Obviously, there can be more than one critical path through the project network and moreover, the prolongation of an activity that does not lie in any critical path may cause that another non-critical path becomes critical. For this reason, the reserve of the path P_i towards the critical path P_c is defined as the difference

$$r(P_i) = t(P_c) - t(P_i)$$

for all paths in the project graph. This reserve determines the possible extension of the path duration that does not cause the path to become a new critical one with a longer duration and consequently, with a longer project duration.

The problem of the realistic estimate of the activities duration was already mentioned as one of the critical problem in project planning and control. In principle, the results of the previous realizations are not available for unique projects and the activity durations must be estimated. This is the typical problem of the development projects or for instance of the software implementation projects. The planning and control of such projects strongly depends on the experience and foresight of the project manager. However, even the project that is similar to some previous one, does not run in exactly the same circumstances. However, the uncertainty into the activities duration setting may be well respected when the deterministic activity durations are substituted by fuzzy quantities. If the activity durations are fuzzy quantities, the path duration is a fuzzy quantity, too, and it can be determined as a fuzzy sum of the durations of particular activities

$$T_s = t_{i_1} \oplus t_{i_2} \oplus \ldots \oplus t_{i_k}$$

With respect to the practical usage, the simplification may be adopted consisting in integer values of the activity durations. Namely, their values may be set as an integer multiple of an appropriately chosen unit of time. Using the relation (4), the membership function of the duration T_s of the path P_s is then expressed by

$$\mu_{T_s}(z) = \sup\,[\min\,(\mu_{i_1}(x_1), \mu_{i_2}(x_1), \ldots, \mu_{i_k}(x_k)) : x_1, x_1, \ldots, x_k \in \mathcal{Z}, x_1 + x_2 + \ldots + x_k = z] \qquad (7)$$

To compare the lengths of particular paths, we use the order relation defined for two fuzzy quantities above. Let P_s and P_r be two paths with durations T_s and T_r, respectively. Then the path P_s is fuzzy longer than the path P_r (i.e. $T_s \geqq T_r$, or simply $P_s \geqq P_r$) with the grade of membership

$$\mu_{T_s \geq T_r}(T_s, T_r)$$

and vice versa, with the grade of membership

$$\mu_{T_r \geq T_s}(T_s, T_r)$$

the path P_r is fuzzy longer than the path P_s, i.e. $T_r \geqq T_s$. As a result of this uncertainty, the critical path is not critical one absolutely but with some grade of membership, and another path is not a critical one with some grade of membership, too.

Let us denote $\mathcal{P}$ the set of all paths in a particular graph. Then the set of all critical paths in this graph is a fuzzy subset of $\mathcal{P}$ and its membership function is given by

$$\mu_C(P_s) = \min\,(\mu_{T_s \geq T_r}(T_s, T_r) : P_r \in \mathcal{P}) \quad \text{for all paths} \quad P_s \in \mathcal{P} \qquad (8)$$

It can be shown – e.g. Mareš (2000) – that there is always at least one path P_s with $\mu_C(P_s) = 1$ and if there are more paths like this one, they all have the same modal value of duration T_s. This path is called the fuzzy critical path P_c. The reserves of other paths are calculated towards this path. It is obvious, that these reserves are fuzzy quantities, too. Fuzzy reserve r_{c-s} of the path P_s towards the fuzzy critical path P_c is given by

$$r_{c-s} = T_c \oplus (-T_s))$$

The membership of the reserve of the path P_s towards the fuzzy critical path P_c is given by

$$\mu_{r_{c-s}}(z) = \sup\,[\min\,(\mu_{T_c}(x), \mu_{T_s}(x-z)) : x \in \mathcal{R}] \quad \text{for all} \ z \in \mathcal{R}. \qquad (9)$$

It follows from these results that with some grade of membership, the fuzzy reserves of some noncritical paths may have negative values. This corresponds to the possibility that some prolongation of an activity lying in a non-critical path causes the prolongation of the whole project. This uncertainty corresponds again very well with the real indeterminateness in true projects.

The uncertainty in the estimates of activity duration may originate from the lack of knowledge or impossibility to predict the realistic values. The described fuzzy approach to the project planning enables to involve not only this uncertainty in the project calculations, but also the existence of the process alternatives with different possibilities of realization. This is another valuable feature of the fuzzy approach to the project planning.

IN THE FRAMEWORK OF UNCERTAINTY THE PROJECT COSTS NEED NOT BE UNCERTAIN

So far we have discussed the project planning and control only from the point of view of the activity and project duration. It is naturally very important. However, meeting the deadline is only one of the project success criteria. With no doubts, the costs control is also very important. Generally, the cost of an activity depends on its duration but the functions describing the relation between activity duration and activity cost may differ for different character of tasks. Hence, it may be helpful to connect the project planning of time and costs together. Moreover, the activity cost depends on the chosen process alternative and on the resources allocation. And as it has been already said, the fuzzy approach makes it possible to describe the uncertainty in project planning also from the uncertainty or vagueness point of view which variant will be chosen.

Let c_i be the cost of the realization of the activity A_i. The cost generally depends on its duration t_i, and let up suppose it does not depend on the duration of any other activity. Since the duration is a fuzzy quantity, the cost is also a fuzzy quantity and according to (5) its membership function is

$$\mu_{c_i}(z) = \mu_{t_i}(t_i) \qquad \text{where } z = c_i(t_i)$$

This relation coincides with the situation we could intuitively expect. It will be necessary to spend the particular cost with the grade of membership that equals the grade of membership of the corresponding activity duration.

Fuzzy approach to the project description allows the project manager to concentrate on the paths that have a higher grade of membership of being critical, and consequently, being dominant for the duration of the whole project. When the costs point of view is added, the project perspective is broadened out by the financial point of view. Of course, the activities with the strongest dependency of cost on duration are focused on primarily, since they may cause the highest saving or loss. And this knowledge is even more valuable when it is connected with the information about the grade of membership, i.e. with the information about the possibility that the particular situations occur.

Project costs are given as a sum of costs of all activities. It is a fuzzy quantity given by

$$C = c_1 \oplus c_2 \oplus \ldots \oplus c_n \quad (10)$$

Its membership function is

$$\mu_C(z) = \sup\ [\min\ (\mu_{t_1}(t_1), \mu_{t_2}(t_2), \ldots, \mu_{t_n}(t_n)) : z = c_1(t_1) + c_2(t_2) + \ldots + c_n(t_n)] \quad (11)$$

The membership function μ_C determines the grade of membership, i.e. the possibility that the project costs value is z. This information can be very important for the project planning. It is a global quantity relevant to the whole project, but to be obtained, it is necessary to know only the duration and cost function of the particular activities. And moreover, there is no need to define these parameters precisely but for every activity it is sufficient just to give the estimates of their possible values with the corresponding grades of membership. Since the activity durations can be taken in fact as integers (as integer multiples of an appropriately chosen unit of time), the evaluation is reduced to the calculation of the finite number of possibilities. The calculations are quite simple and in spite of the great number of activities in the real life projects, they can be carried out by the present information technologies without any problem.

VARIANTS OF PROJECT SOLUTION

Planning the project, the project manager has often to compare several variants (alternatives) of the problem solution. The project time or costs belong to the commonly used criteria for such comparison. The activities time or their costs are usually not precisely known in the planning phase, but to be able to compare the variants, it is usually sufficient to know the estimates of the possible values of these quantities and their corresponding grades of membership.

Let the set of all possible (compared) variants of project solutions be

$$\boldsymbol{\mathcal{V}} = \{V_1, V_2, \ldots, V_M\} \qquad \text{where } M \text{ is a natural number}$$

and let C_s be the costs of the variant V_s, $s = 1, 2, \ldots, M$. According to the theory mentioned above, these costs are fuzzy quantities, too, and we can use the introduced order relation of fuzzy quantities (6).

The costs C_s of a variant V_s are fuzzy greater than the costs C_r of the variant V_r, $C_s \geqq C_r$, with the membership function

$$\mu_{C_s \geq C_r}(C_s, C_r)$$

and the costs C_r of a variant V_r are fuzzy greater than the costs C_s of the variant V_s, $C_r \geqq C_s$, with the membership function

$$\mu_{C_r \geq C_s}(C_s, C_r)$$

Like in the case of the critical path, it is not possible to determine the variant with the lowest costs uniquely, but only with a certain grade of membership.

The set of all variants with the lowest costs is a fuzzy set with the membership function

$$\mu_L(V_s) = \min\ (\mu_{C_s \geq C_r}(C_s, C_r) : V_r \in \boldsymbol{\mathcal{V}}) \quad \text{for all variants } V_s \in \boldsymbol{\mathcal{V}}$$

And similarly like in the case of the critical path, there is at least one variant V_s with $\mu_C(V_s) = 1$, and if there are more than one such variants, the modal value of all of them will be the same. We called this variant the fuzzy cheapest one and denote it by V_L. The cost difference between each variant and the fuzzy cheapest variant may be set as well. It is obvious that the cost differences are fuzzy quantities. Let d_s be the cost difference between the variant V_s and the variant V_L. It may be calculated as

$$d_s = C_s \oplus (-C_L)$$

and its membership function is

$$\mu_{d_s}(z) = \sup\ [\min\ (\mu_{C_s}(x), \mu_{C_L}(x-z)) : x \in \boldsymbol{\mathcal{R}}] \quad \text{for all } z \in \boldsymbol{\mathcal{R}}$$

Hence, even for variants that are not the fuzzy cheapest, the fuzzy costs difference, when the variant is compared with the fuzzy cheapest variant, may be negative with a certain grade of membership. It means that with some possibility the costs of variants that are not fuzzy cheapest may be even lower than the costs of the fuzzy cheapest variant.

The knowledge of the grade of possibility that some variant is cheaper than another one gives a valuable basis for the decision between the variants and it corresponds very well with the uncertainty recognized in the real project planning.

EXAMPLE

To illustrate how the theory introduced above may be used in the phase of project planning, the following example of a project with three possible variants is described. It is not a real life project but only a dummy project with a very simple structure. It enables to follow the calculations sequence and its consequences and to clarify the benefits of the used technique.

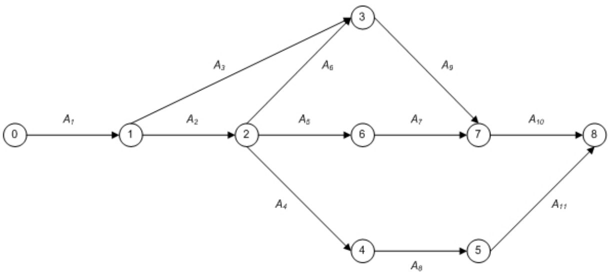

Figure 1. Variants V_1 and V_2 – project workflow

Project variant V_1

Figure 1 graphically shows the workflow of the project variant V_1. The numbered vertices represent the project states, the directed edges marked A_i , $i = 1, 2, ..., 11$, stand for the project activities. The project beginning (in the state 0), the project end (in the state 8) and all relationships among the project activities are obvious from Figure 1 (e.g. the activity A_9 can start only after the activities A_3 a A_6 are finished).

Table 1 brings together the project activities A_i, $i = 1, 2, ..., 11$, the values of their membership functions $\mu_i(t_i)$ for their duration t and their costs functions c_i as the functions of activities durations. The time unit of the activity durations is arbitrary but the same for all values, the constants used in cost functions are related to arbitrary but the same currency unit.

It is obvious from Table 1 that in this project example, the activity durations are expected to be rather longer than shorter than the activity durations with the highest grade of membership. Note that the activity A_{11} can be prolonged only by two time units (the prolongation by one time unit is not possible) in this project example. In the real life projects, it may be caused for example by technological requirements. Costs functions depend generally on the activity durations. In the project example, the dependence of costs functions on the activity durations is linear but in fact, it could be quite general. The theory introduced above does not restrict the character of the dependence. This simplification was used only to better illustrate the results. Some cost functions (like for the activity A_1) include constant values connected for example with the activity beginning in the real project. Some cost functions (like for the activities A_9, A_{10}) include additional costs applicable if the activity duration exceeds some time limit.

It is obvious from the Figure 1 that there are four possible paths in the project variant V_1

$P_1 = \{A_1, A_3, A_9, A_{10}\}$

$P_2 = \{A_1, A_2, A_6, A_9, A_{10}\}$

$P_3 = \{A_1, A_2, A_5, A_7, A_{10}\}$

$P_4 = \{A_1, A_2, A_4, A_8, A_{11}\}$

According to (7), the membership functions of the duration of these four paths are given by the Table 2.

Table 1. Variant V_1 – project activities, their membership functions and costs functions

Activity	Membership function						Costs function
A_1	$\mu_1(2) = 0.2$	$\mu_1(3) = 1$	$\mu_1(4) = 0.5$	$\mu_1(5) = 0.2$		$\mu_1(t_1) = 0$ elsewhere	$c_1(t_1) = 1000 + 800 \times t_1$
A_2	$\mu_2(3) = 0.1$	$\mu_2(4) = 1$	$\mu_2(5) =0 .3$			$\mu_2(t_2) = 0$ elsewhere	$c_2(t_2) = 1000 \times t_2$
A_3		$\mu_3(7) = 1$	$\mu_3(8) = 0.5$	$\mu_3(9) = 0.2$		$\mu_3(t_3) = 0$ elsewhere	for $t_3 \leq 7$: $c_3(t_3) = 1000 \times t_3$ for $t_3 > 7$: $c_3(t_3) = 1000 \times t_3 + 500 \times (t_3 - 7)$
A_4	$\mu_4(6) = 0.1$	$\mu_4(7) = 1$	$\mu_4(8) = 0.3$	$\mu_4(9) = 0.1$		$\mu_4(t_4) = 0$ elsewhere	$c_4(t_4) = 800 \times t_4$
A_5		$\mu_5(2) = 1$	$\mu_5(3) = 0.2$			$\mu_5(t_5) = 0$ elsewhere	$c_5(t_5) = 700 \times t_5$
A_6	$\mu_6(5) = 0.3$	$\mu_6(6) = 1$	$\mu_6(7) = 0.4$	$\mu_6(8) = 0.1$		$\mu_6(t_6) = 0$ elsewhere	$c_6(t_6) = 500 + 700 \times t_6$
A_7	$\mu_7(4) = 0.2$	$\mu_7(5) = 1$	$\mu_7(6) = 0.8$	$\mu_7(7) = 0.3$		$\mu_7(t_7) = 0$ elsewhere	$c_7(t_7) = 1000 \times t_7$
A_8	$\mu_8(2) = 0.3$	$\mu_8(3) = 1$	$\mu_8(4) = 0.8$	$\mu_8(5) = 0.6$	$\mu_8(6) = 0.1$	$\mu_8(t_8) = 0$ elsewhere	$c_8(t_8) = 800 \times t_8$
A_9		$\mu_9(5) = 1$	$\mu_9(6) = 0.5$	$\mu_9(7) = 0.4$	$\mu_9(8) = 0.2$	$\mu_9(t_9) = 0$ elsewhere	for $t_9 \leq 6$: $c_9(t_9) = 1000 \times t_9$ for $t_9 > 6$: $c_9(t_9) = 1000 \times t_9 + 2000$
A_{10}	$\mu_{10}(3) = 0.1$	$\mu_{10}(4) = 1$	$\mu_{10}(5) = 0.5$	$\mu_{10}(6) = 0.1$		$\mu_{10}(t_{10}) = 0$ elsewhere	for $t_{10} \leq 5$: $c_{10}(t_{10}) = 1000 \times t_{10}$ for $t_{10} > 5$: $c_{10}(t_{10}) = 1000 \times t_{10} + 500 \times (t_{10} - 5)$
A_{11}		$\mu_{11}(4) = 1$		$\mu_{11}(6) = 0.7$		$\mu_{11}(t_{11}) = 0$ elsewhere	$c_{11}(t_{11}) = 900 \times t_{11}$

Table 2. Variant V_1 – membership functions of the paths duration

	Duration of the path																		
	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
μ_{T_1}	0.0	0.0	0.0	0.1	0.2	1.0	0.5	0.5	0.5	0.5	0.4	0.2	0.2	0.2	0.1	0.0	0.0	0.0	0.0
μ_{T_2}	0.0	0.0	0.0	0.0	0.1	0.1	0.2	0.3	1.0	0.5	0.5	0.5	0.4	0.4	0.3	0.2	0.2	0.2	0.1
μ_{T_3}	0.1	0.1	0.2	0.2	1.0	0.8	0.5	0.5	0.3	0.3	0.2	0.2	0.1	0.0	0.0	0.0	0.0	0.0	0.0
μ_{T_4}	0.0	0.0	0.0	0.1	0.1	0.2	0.3	1.0	0.8	0.7	0.7	0.6	0.5	0.3	0.3	0.2	0.1	0.1	0.0

Table 3. Variant V_1 – membership functions value of paths comparison and grade of membership every path becomes critical with

$\mu_{T_1 \geq T_2} = 0.5$	$\mu_{T_1 \geq T_3} = 1$	$\mu_{T_1 \geq T_4} = 0.5$	$\mu_C(P_1) = 0.5$
$\mu_{T_2 \geq T_1} = 1$	$\mu_{T_2 \geq T_3} = 1$	$\mu_{T_2 \geq T_4} = 1$	$\mu_C(P_2) = 1$
$\mu_{T_3 \geq T_1} = 0.8$	$\mu_{T_3 \geq T_2} = 0.3$	$\mu_{T_3 \geq T_4} = 0.5$	$\mu_C(P_3) = 0.3$
$\mu_{T_4 \geq T_1} = 1$	$\mu_{T_4 \geq T_2} = 0.8$	$\mu_{T_4 \geq T_3} = 1$	$\mu_C(P_4) = 0.8$

Table 4. Variant V_1 – project duration

	Project duration																		
	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
T	0.0	0.0	0.0	0.0	0.1	0.1	0.2	0.3	1.0	0.7	0.7	0.6	0.5	0.4	0.3	0.2	0.2	0.2	0.1

Now we compare the durations of all paths. It means that we determinate the grade of membership of the situation when a path is longer than another one. According to (8), we can also calculate for every path with what grade of membership it becomes critical.

Table 3 shows that the path P_2 is the longest and thus it is the critical path with the highest grade of membership. The path P_4 may become critical with a quite high grade of membership (0.8). Hence, it is very important to pay attention not only to the activities in the path P_2 but also to the activities in the path P_4. This is the benefit from the fuzzy approach to the project planning. We have the information concerning not only the critical path, but also the grade of membership which another path may become critical with.

Finally, we calculate the membership function of the project duration T using the relations (1), (2) and (6).

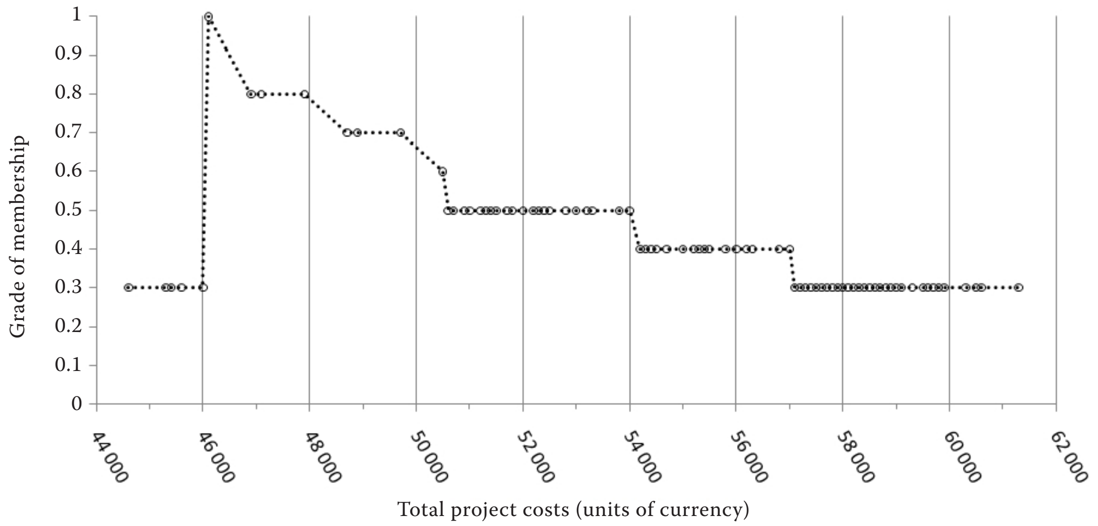

Figure 2. Project costs – variant V_1

As it can be seen from Table 4, the project duration is 22 time units with the highest grade of membership. With the quite high grade of membership of 0.7 and more, the project may take up 2 time units more, and with the grade of membership of 0.5 and more, the project may take up 4 time units more. On the other hand, the project may be shorter than 22 time units with much a smaller grade of membership – the project may be 1 time unit shorter with the grade of membership of 0.3.

The dominant influence of the membership function values μ_{T_2} of the critical path P_2 on the grade of membership values given in Table 4 can be observed. Nevertheless, due to the high grade of membership, the path P_4 may become critical and the influence of the membership function values μ_{T_4} is well observable, too.

Now, the time perspective will be enriched by the cost point of view. The total project costs amount as the sum of all project activities costs is a fuzzy quantity described by the relation (10). Its membership function is given by (11). Figure 2 shows the grade of membership of some possible project costs of the project variant V_1.

Note that the values of the grade of membership smaller than 0.3 are omitted due to the lucidity of the graph. Analogically, lower values of the grade of membership are not displayed if higher costs have a higher grade of membership. The dashed polyline in Figure 2 connects the maximum amounts of the grade of membership and represents the pessimistic estimate of the grade of membership of the total costs in the project variant V_1.

The highest grade of membership corresponds to the total costs of 46 100 units of currency. Moreover, it is obvious from Figure 2 that the total costs amounting higher than this value (up to the value of 57 000 units of currency) have a higher grade of membership than even a little bit lower total costs amounts.

Project variant V_2

The workflow of the project variant V_2 can be graphically described by the same scheme as the project variant V_1 – see Figure 1. However, the technological processes of activities A_8 a A_{11} differ from the processes used in the project variant V_1 and consequently, the activities A_8 and A_{11} take a longer time but have lower costs when comparing the project variant V_2 to the project variant V_1. The membership function and the costs function of these two activities are shown in Table 5. The membership functions and the costs functions of other activities are the same in both project variants and are listed in Table 1.

All project paths have the same definition in both project variants V_1 and V_2. Since the technological changes do not influence the paths P_1, P_2 and P_3, the membership functions for their durations are the same in both project variants (Table 2). The membership function for the duration of the path P_4 is given in Table 6.

Further, we determine the grade of membership of the situation when a path is longer than another one, and for every path we calculate with what grade of membership it becomes the critical path (Table 7).

We have found that the path P_4 is the longest one and thus it is the critical path with the highest grade of membership. Other paths may become critical with a significantly lower grade of membership in the project variant V_2.

The membership function of the project duration T is given by Table 8 in the project variant V_2.

It follows from the Table 8 that the project variant V_2 results in a project prolongation. The project duration is 24 time units with the highest grade of membership (it is 22 time units in the variant V_1). The grade of membership of an eventual project prolongation above 24 time units is lower than in the project variant V_1 – it is only 0.5 or less. The

Table 5. Variant V_2 – project activities (different from the variant V_1), their membership functions and costs functions

Activity	Membership function					Costs function
A_8	$\mu_8(5) = 0.2$	$\mu_8(6) = 1$	$\mu_8(7) = 0.4$	$\mu_8(8) = 0.1$	$\mu_8(t_8) = 0$ elsewhere	$c_8(t_8) = 500 \times t_8$
A_{11}	$\mu_{11}(3) = 0.2$	$\mu_{11}(4) = 1$	$\mu_{11}(5) = 0.3$		$\mu_{11}(t_{11}) = 0$ elsewhere	$c_{11}(t_{11}) = 600 \times t_{11}$

Table 6. Variant V_2 – membership functions of the path P_4 duration

	Duration of the path																		
	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
μ_{T_4}	0	0	0	0	0	0.1	0.1	0.2	0.2	0.2	1.0	0.5	0.4	0.3	0.3	0.3	0.2	0.1	0.1

Table 7. Variant V_2 – membership functions value of paths comparison and grade of membership every path becomes critical with

$\mu_{T_1 \geq T_2} = 0.5$	$\mu_{T_1 \geq T_3} = 1$	$\mu_{T_1 \geq T_4} = 0.4$	$\mu_C(P_1) = 0.4$
$\mu_{T_2 \geq T_1} = 1$	$\mu_{T_2 \geq T_3} = 1$	$\mu_{T_2 \geq T_4} = 0.5$	$\mu_C(P_2) = 0.5$
$\mu_{T_3 \geq T_1} = 0.8$	$\mu_{T_3 \geq T_2} = 0.3$	$\mu_{T_3 \geq T_4} = 0.2$	$\mu_C(P_3) = 0.2$
$\mu_{T_4 \geq T_1} = 1$	$\mu_{T_4 \geq T_2} = 1$	$\mu_{T_4 \geq T_3} = 1$	$\mu_C(P_4) = 1$

Table 8. Variant V_2 – project duration

	Project duration																		
	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
T	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.2	0.2	0.2	1.0	0.5	0.4	0.4	0.3	0.3	0.2	0.2	0.1

project may be longer by 1 time unit with the grade of membership equal to 0.5, and it may be shorter by 3 time units with the grade of membership equal to only 0.2 .

The dominant influence of the membership function values μ_{T_4} of the critical path P_4 on the grade of membership values given in Table 8 is obvious. In this variant, the grade of membership other paths may become critical with is rather low and the influence of the membership function values of other paths is almost negligible.

Similarly like for the project variant $V_{1,}$ the possible costs of the project variant V_2 are determined together with their grades of membership.

The highest grade of membership corresponds to the total costs amount of 45 500 units of currency in the project variant V_2. It can be seen from Figure 3 that in the project variant V_2, like in the project variant V_1, the costs which are a little bit higher than this amount have a much higher grade of membership than the costs a little bit lower than this amount. The total costs amounting from 45 500 to 55 000 units of currency have the grade of membership higher than 0.3 in the project variant V_1. Generally, the total project costs of the project variant V_2 are supposed to be lower than the total project costs of the project variant V_1.

Project variant V_3

Another technological changes are introduced in the project variant V_3. The activities A_8 and A_{11} are substituted by the activity A_{12}. Then the workflow of the project variant V_3 is described graphically in Figure 4.

The membership function and the costs function of the activity A_{12} is given by Table 9. The membership functions and costs functions of other activities in the project variant V_3 are the same as those in

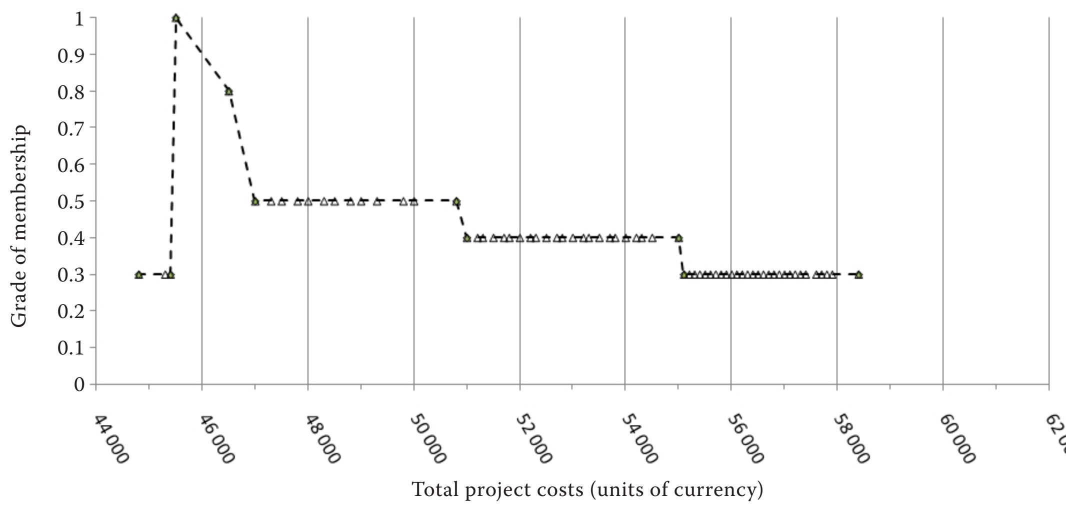

Figure 3. Project costs – variant V_2

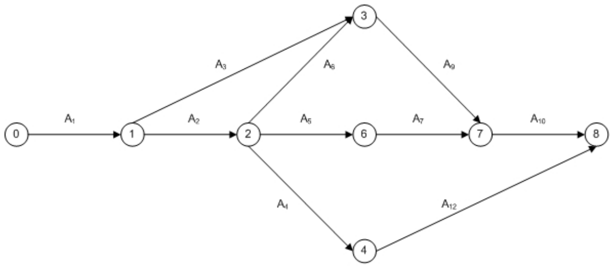

Figure 4. Variant V_3 – project workflow

the project variant V_1 and they are listed in Table 1 (except activities A_8 and A_{11} which are not relevant in the project variant V_3).

Project paths P_1, P_2 and P_3 have the same definition in all project variants V_1, V_2 and V_3. Since the technological changes do not influence the paths P_1, P_2 and P_3, the membership functions for their durations are the same in all project variants (see Table 2).

The project paths P_4 is given by

$P_4 = \{A_1, A_2, A_4, A_{12}\}$

and the membership function for the duration of the path P_4 is given in the Table 10.

Similarly like for the previous project variants, we determine the grade of membership of the situation when a path is longer than another one and for every path we calculate with what grade of membership it becomes the critical one (Table 11).

Evidently, the path P_2 is the longest one and thus it is the critical path with the highest grade of membership in the project variant V_3. Other paths may become critical with the grade of membership of 0.5 or lower. Accordingly, the path P_4 may become critical with a lower grade of membership in the project variant V_3 than in the project variant V_1.

In the project variant V_3, the membership function of project duration T is given in Table 12.

The Table 12 shows that the project duration T with the highest grade of membership is 22 time units in the project variant V_3 as well as in the project variant V_1. In the project variant V_3, like in the variant V_1, the grade of membership of the project duration longer than 22 time units is only 0.5 or less. The interval of values with the grade of membership 0.5 or higher is wider than in the variant V_2 but narrower than in the variant V_1. In the project variant V_3, the project duration may be up to 3 time units longer than the value with the highest grade of membership. In all variants, the project duration shorter than the value with the highest grade of membership has only low values of the

Table 9. Variant V_3 – project activity (different from the variant V_1), its membership function and costs function

Activity	Membership function				Costs function
A_{12}	$\mu_{12}(6) = 0.6$	$\mu_{12}(7) = 1$	$\mu_{12}(8) = 0.2$	$\mu_{12}(t_{12}) = 0$ elsewhere	$c_{12}(t_{12}) = 1000 \times t_{12}$

Table 10. Variant V_3 – membership functions of the path P_4 duration

	Duration of the path																		
	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
μ_{T_4}	0	0	0	0.1	0.1	0.2	0.6	1.0	0.5	0.3	0.3	0.2	0.2	0.1	0	0	0	0	0

Table 11. Variant V_3 – membership functions value of paths comparison and grade of membership every path becomes critical with

$\mu_{T_1 \geq T_2} = 0.5$	$\mu_{T_1 \geq T_3} = 1$	$\mu_{T_1 \geq T_4} = 0.5$	$\mu_C(P_1) = 0.5$
$\mu_{T_2 \geq T_1} = 1$	$\mu_{T_2 \geq T_3} = 1$	$\mu_{T_2 \geq T_4} = 1$	$\mu_C(P_2) = 1$
$\mu_{T_3 \geq T_1} = 0.8$	$\mu_{T_3 \geq T_2} = 0.3$	$\mu_{T_3 \geq T_4} = 0.5$	$\mu_C(P_3) = 0.3$
$\mu_{T_4 \geq T_1} = 1$	$\mu_{T_4 \geq T_2} = 0.5$	$\mu_{T_4 \geq T_3} = 1$	$\mu_C(P_4) = 0.5$

Table 12. Variant V_3 – project duration

	Project duration																		
	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
T	0	0	0	0	0.1	0.1	0.2	0.3	1.0	0.5	0.5	0.5	0.4	0.4	0.3	0.2	0.2	0.2	0.1

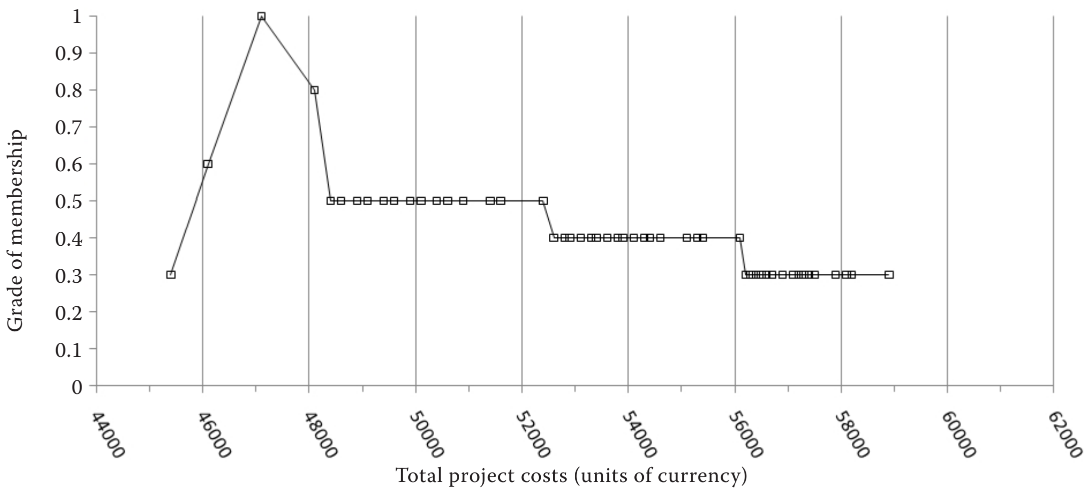

Figure 5. Project duration – variants V_3

grade of membership – the grade of membership of the project duration 1 time unit shorter is only 0.3.

Again, the dominant influence of the membership function values μ_{T_2} of the critical path P_2 on the grade of membership values given in the Table 12 is evident. In this variant, like in the variant V_2, the grade of membership other paths may become critical with is rather low and the influence of the membership function values of other paths is negligible.

The possible costs of the project variant V_3 with the corresponding grades of membership are shown in Figure 5. In the project variant V_3, the highest grade of membership corresponds to the total costs amount of 47 100 units of currency. This value is the highest one from all three project variants. In all three variants, the total project costs a little bit higher than the value with the highest grade of membership have a higher grade of membership than the total project costs a little bit lower than the value with the highest grade of membership. Nevertheless, only in the project variant V_3 the total costs are lower than the value with the highest grade of membership which have the grade of membership higher than 0.3. The total costs amounting from 46 100 to 56 100 units of currency have the grade of membership higher than 0.3 in the project variant V_3. This is almost the same interval like in the project variant V_1, but the total costs with the highest grade of membership are higher in the project variant V_3 than in the project variant V_1. Hence, the project variant V_3 is connected with a significant risk of higher project costs than in the case of the project variant V_1.

Comparison of variants V_1, V_2 and V_3

The project example is very simple. However, it is obvious how the fuzzy approach enriched by the costs point of view can extend the knowledge available during the project planning process.

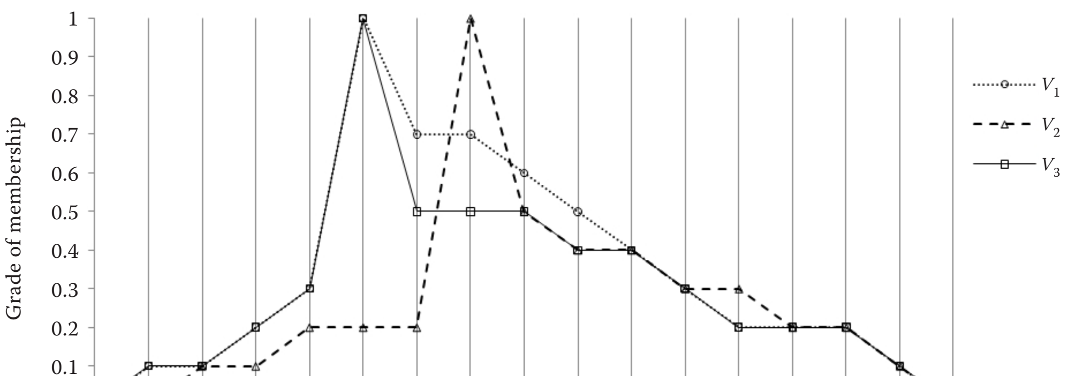

Figure 6. Project duration – variants V_1, V_2, V_3

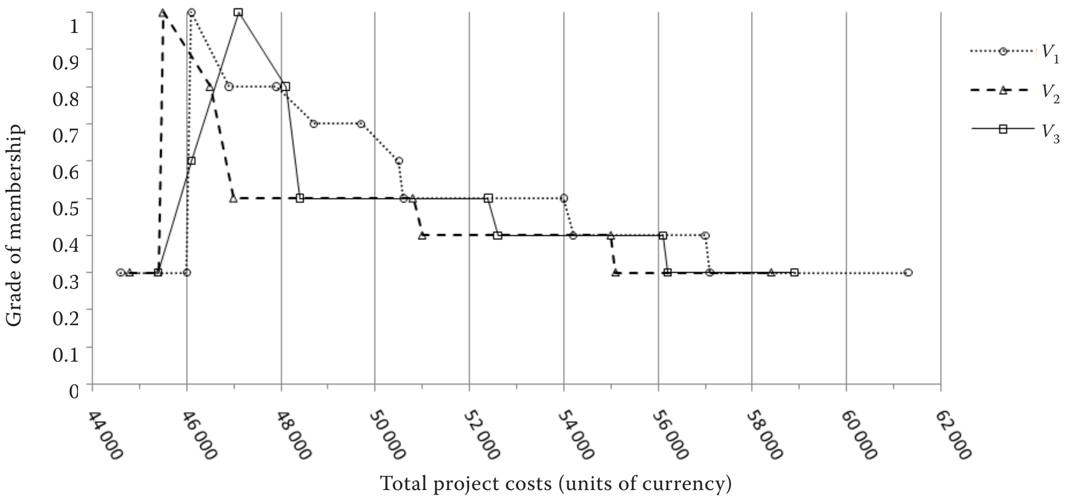

Figure 7. Project costs – variants V_1, V_2, V_3

Figure 6 compares all three variants from the project duration point of view.

The project duration with the highest grade of membership is the same in the project variants V_1 and V_3 and it is 2 time units shorter than the project duration with the highest grade of membership in the project variants V_2. The risk of an eventual project prolongation (up to 4 time units) is slightly higher in the project variant V_1 than in the project variant V_3.

Figure 7 compares all three variants from the total project costs point of view.

From the point of view of the total project costs with the highest grade of membership, the cheapest one is the variant V_2, the project variant V_1 lies in the middle and the project variant V_3 is the most expensive. In all three variants, there is a significant risk of higher project costs as compared to the value with the highest grade of membership, whereas lower project costs (as compared to the value with the highest grade of membership) have only a small grade of membership. Only in the project variant V_3 the lower values of project costs have a little bit higher grade of membership, and it is true only for the project costs value with the highest grade of membership in the project variant V_1.

In another words, if the costs minimisation is the main criterion for the decision between variants and a slightly longer project duration is acceptable, the best solution seems to be the project variant V_2. On the other hand, if the project duration is crucial, the project variants V_1 and V_3 are more acceptable and with respect to the higher project costs of the variant V_3, the variant V_1 seems to be the better solution. However, in the project variant V_1, the grade of membership of project prolongation is higher than in the project variant V_3 and this risk should be taken into account in the project planning and managing process.

CONCLUSIONS

The fuzzy approach to the modelling of uncertainty in project description extends our possibilities and enables to introduce more realistic project characteristics. It makes it possible to complete the description of the supposed project process used in the classical deterministic methods with the possible deviations and variations including the quantification of the possibility of their appearance. This information broadening is extremely important for the decision making which unavoidably accompanies every phase of the project planning and managing.

The existing fuzzy methods are based on the fuzzy approach in time scheduling of the project workflow. In the article, this fuzzy approach is improved by the addition of the project costs perspective to the standard time aspect. The benefits of such improvement are illustrated by an example of project variants. The information concerning the possibility with which the project duration and project costs may become lower or higher can be crucial for a good decision making and project management. This information may also serve as a valuable guide for decisions which activities or project parts should be managed preferentially to keep the project dates and costs under control.

Even though the used mathematical operations are very simple, the increasing project size causes the rapid growth of the possible values to be compared and processed and it is necessary to use the information technologies to solve the problems of the real life projects. It is also suitable to use the techniques of sequential refinement when the starting rough estimates are subsequently more precisely specified. The advantages of this technique are evident in extremely large projects. In the first approximation, the project is divided into blocks ("macro activities") which enable to set or at least to estimate their possible duration and the dependences of their costs on the duration. Then the grade of membership is set for all possible values of each "macro activity" duration. Also some alternatives or variants may be included into the project description. The possible project duration and the corresponding costs are calculated and analysed. If necessary, these steps can be repeated on the individual "macro activity" or the project variant divided into more detailed activities.

The critical path control without the parallel cost evaluation corresponds to a very rare case of project when the project costs do not matter. However, in most projects the project costs belong to the key parameters and they are carefully monitored. Such projects can benefit from the suggested approach with the parallel control of activities on the paths with the high grade of membership to become critical and the project costs control with the possibility quantification of spending them. This approach can be used in the project planning phase and the evaluation of possible variants, in the phase of risk definition and evaluation as well as in project and risk management. It may improve the decision making process and reduce the risk to a reasonable level. The method described above is generally applicable and can serve as an effective management tool in an arbitrary business sphere and in an enterprise of an arbitrary size. As such, it may also contribute to the increasing competitiveness of companies operating in the current dynamic environment of agrarian market.

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