The impact of credit rationing on farmer's economic equilibrium

Vliv úvěrového omezení na farmářovu ekonomickou rovnováhu

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Abstract: The paper deals with the theoretical analysis of the impact of credit rationing on farmer's economic equilibrium. The analysis is carried out based on the derived dynamic optimization model, which is the dynamic investment model with adjustment costs. The credit rationing is introduced by imposing an upper limit on the control variable, which is in this case represented by the investment spending. Then, the optimal control is used to solve the optimization problem in the situation of both with and without credit constraints. Finally, the situations without and with credit rationing are compared. The results show that the occurrence of credit rationing or in general financial constraints significantly determines the capital accumulation and investment decisions of farmers and as a result their supply functions.

Key words: credit rationing, farmer, economic equilibrium, optimal control, adjustment costs

Abstrakt: Článek se zabývá teoretickou analýzou vlivu úvěrového omezení na ekonomickou rovnováhu zemědělce. Analýza je založena na odvozeném dynamickém investičním modelu s náklady přizpůsobení. Úvěrové omezení je do modelu zaneseno horním omezením na kontrolní proměnnou, která je reprezentována investicemi. Řešení dynamického optimalizačního problému je založeno na metodě optimální kontroly. Obdržené výsledky ukazují, že výskyt úvěrového omezení, resp. obecně finančního omezení signifikantně determinuje zemědělcovu akumulaci kapitálu, investiční rozhodování a jeho nabídkovou funkci.

Klíčová slova: úvěrové omezení, zemědělec, ekonomická rovnováha, optimální kontrola, náklady přizpůsobení

Financial constraint is often discussed problem the farmers must face. One of the sources of this problem is imperfect capital market or the phenomenon of credit rationing, respectively. The paper attempts to show the impact of credit rationing or in general financial constraint on farmer's economic equilibrium, respectively. The analysis is theoretical and it is based on dynamic investment adjustment cost model. The solution of the dynamic optimisation problem is the main contribution of the paper that helps to understand better the behaviour of economic agents compared to static analysis[1].

Credit rationing in Czech agriculture has been theoretically analyzed by Janda (1994, 2002) and both theoretically and empirically – by Čechura (2005, 2008a, b). The activities and the role of SGAFF (Supporting and Guarantee Agricultural and Forestry Fund) in reducing credit rationing problem in Czech agriculture was further analyzed by e.g. Bečvářová (2006), Čechura (2006), Janda, Čajka (2006), Janda (2006), Šilar (1995). The authors agree that SGAFF makes agricultural credit accessible, although several aspects of their activities were criticized. Broader analysis of the economic performance of Czech farmers can be

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[1]The paper is based on Čechura (2008a).

found in Střeleček et al. (2007), Froněk et al. (2007), Střeleček et al. (2008), etc.

AIMS AND METHODOLOGY

The aim of this paper is to analyze the impact of credit rationing on the economic equilibrium of farmers. The theoretical analysis should answer the following question: What is the potential effect of the credit rationing on the farmer's economic equilibrium or, more specifically, on farmer's capital accumulation and supply function, respectively?

The hypothesis of the paper is: The occurrence of credit rationing in the agricultural loan market significantly determines the capital accumulation and investment decisions of farmers and as a result their supply functions.

As we deal with credit rationing and economic equilibrium, we must define how these two terms are employed for our purposes. Credit rationing is a situation where a farmer or a group of farmers apply for a loan but do not receive it or do not receive the demanded amount. The farmer's economic equilibrium in this paper is a state when the farmer follows the optimal path of capital, investment and supply without financial constraint during the time period.

The theoretical model is defined in the form of a dynamic optimization model. The derived model stems from the simple investment model. The paper is innovative because of the introduction of upper constraints on the control variable, which represents the occurrence of credit rationing, and solution and analysis of the paths of capital, investment and supply in the situation of financial constraints under defined scenarios. The method of optimal control is used to solve the optimization problem.

RESULTS AND DISCUSSION

Theoretical model

The derivation of the theoretical model stems from the following assumptions: (i) Loans are an important part of the financing of investment; and (ii) Credit rationing determines the operational activity only through the investments.

Then, the theoretical model is based on a simple optimal dynamic investment model in which the farmers solve the investment problem. However, in spite of the simple specification of the model, an upper limit on the investment was introduced and, thus, the credit rationing or in general financial constraint, respectively, can be analyzed without explicit modeling the farmer's financial situation. This is the subject of many papers on this topic (as mentioned e.g. Schworm 1980; Steigum 1983). The basic features of the results are the same.

It is assumed in the model that economic agents (in this case farmers) are rational. The economic agents base their business decisions on the solution of dynamic optimization problem in an infinite horizon. The model is general enough to comply with the characteristics of small farmers, as well as mid-sized and large agricultural enterprises. This feature of the model is very important because empirical analyses show that the aggregate supply in the Czech agriculture is significantly heterogeneous as far as the economic characteristics of economic agents are concerned (see e.g. Čechura 2005).

Each farmer is endowed with capital k_0 and technology z at the beginning of the period, i.e. in time $t = 0$. It is assumed from the nature of the model that the non-negativity constraint on the capital $k_t \geq 0$ is not binding in the interval $t \in (0;\infty)$. No assumption is placed on the terminal value of the capital, i.e. on $\lim_{t\to\infty} k_t$. The capital is employed in production, namely to produce output y_t. The transformation of the capital into the output is described (for the simplicity of exposition but without loss of generality) by the Cobb-Douglas production function, $y_t = \alpha k_t^{\beta} z l_t^{1-\beta}$, with technology z and labor l_t. Technology is incorporated into the production function as a coefficient. It is assumed that the change of technology, which assures the farmers' competitiveness on the agricultural market for a given time period, is represented by a shift in the parameter z. The shift (increase) of the parameter z causes production to be more productive, i.e. it causes an upward shift of the production function. Labor is normalized to one without loss of generality, i.e. the production function can be written as $y_t = \alpha k_t^{\beta} z$. The production function is differentiable, strictly increasing and concave. Total profit, π_t, which is to be maximized, is the difference between the value of output, $p_t y_t$, and the investment cost. The price is assumed to be exogenously given and its variation does not influence the farmers' decision. However, the uncertainty might be incorporated by letting p_t follow the stochastic process (see e.g. Abel, 1983). Investment costs are given by (1). That is, the farmer undertakes gross investment by incurring an increasing strictly convex cost of adjustment $c(I_t)$.

$$c(I_t) = \rho I_t + \sigma I_t^2 \tag{1}$$

The investment is financed from additional resources, especially from retain earnings and loans.

Moreover, the farmer discounts his profit at a constant rate $r > 0$.

It follows from the nature of the model that we can speak about the decision process of one farmer instead of all farmers without loss of generality. Thus, the result for one farmer also holds for other farmers till it is said otherwise.

Capital accumulation follows the differential equation $dk_t = (I_t - bk_t)dt$, where b states for a constant proportional rate of physical depreciation.

The financing of capital and investments is not modeled but is implicitly incorporated into the model by the introduction of the upper constraint on the investment I_t. Thus, the gross investment has both upper and lower bound. The upper bound represents the financial constraint, the credit rationing phenomenon as defined in methodology, of the farmer in time t. That is, since financial constraint is in our model caused by the occurrence of credit rationing that the farmer faces, these two terms are used interchangeably in following sections.

It follows from the model definition that credit rationing determines only investment financing. It is assumed that the capital k_t is always available. In other words, as the capital k_t is the sum of the loans L_t and the equity E_t, $k_t = L_t + E_t$, which states for the balance sheet identity, then the volume of the loan L_t, if $L_t > 0$, is available. The failure of operational financing due to credit rationing is not the subject of the analysis.

To sum up, the farmer wants to maximize profit or the value of the firm (in this representation), respectively, subject to the state variable accumulation and its initial value, and the control variable constraint. The state variable is the capital (k_t) employed in the production of n agricultural outputs and the control variable is investment (I_t). Time is infinite. Thus, the problem can be written as follows.

$$\max_{I(\cdot)} \int_0^\infty e^{-rt}[p_t \alpha k_t^\beta z - c(I_t)]dt \tag{2}$$

subject to

$$dk_t = (I_t - bk_t)d\,,\ k(0) = k_0 \tag{3}$$

$$0 \le I_t \le B_t \tag{4}$$

where e^{-rt} is used to stand for discounting[2] with a discount interest rate r. B_t stands for financial constraint in time t.

Since the price p_t is exogenously given and its variation does not influence farmer's decision, so for simplicity of the exposition it is assumed that price is constant for the optimization horizon, i.e. from now on the price is incorporated as a parameter into the model. It can be assumed in this situation that the farmer follows the price expectation based on the simple adaptive expectation.

We use a current value Hamiltonian to solve the problem. Employing the current value Hamiltonian we get an autonomous set of equations, which is easier to solve because it results in a pair of autonomous differential equations.[3]

The current value Hamiltonian for our problem (2)–(4) is as follows:

$$\mathcal{H}(k,I,m;\chi) = p\alpha k_t^\beta z - c(I_t) + m(I_t - bk_t)$$

$$\text{subject to } 0 \le I_t \le B_t \tag{5}$$

where m is the current value multiplier.[4]

As the control variable is bounded ($0 \le I_t \le B_t$), we use the Kuhn-Tucker conditions to solve the problem. Thus, we append the constraints to the objective with multiplier w_1 and w_2. The resulting Langrangian for (5) is:

$$L = \mathcal{H} + w_{1t} I_t + w_{2t}(B_t - I_t)$$

that is

$$L = p\alpha k_t^\beta z - \rho I_t - \sigma I_t^2 + m(I_t - bk_t) + w_{1t} I_t + w_{2t}(B_t - I_t) \tag{6}$$

From (6) we may obtain the necessary conditions for the solution of a constrained maximum with respect to I_t:

$$m_t' = m_t(r+b) - p\alpha\beta k_t^{\beta-1} z \tag{7}$$

$$k_t' = I_t - bk_t \tag{8}$$

and the optimality condition

$$\partial L / \partial I = -\rho - 2\sigma I_t + m_t + w_{1t} - w_{2t} = 0 \tag{9}$$

$$w_{1t} \ge 0,\ w_{1t} I_t \ge 0;\ w_{2t} \ge 0;\ w_{2t}(B_t - I_t) = 0 \tag{10}$$

[2] $\lim_{t\to\infty}(1 - r/n)^{nt} = e^{-rt}$, where the interest rate r is compounded n times per year.

[3] The current value Hamiltonian is defined as $\mathcal{H} \equiv e^{-rt} H = f(t,k,I) + mg(t,k,I)$.

[4] $m(t) \equiv e^{rt}\lambda(t)$; λ_t is the marginal value of the state at t, which is discounted back to time zero, whereas the current value multiplier $m(t)$ gives the marginal value of the state variable at time t in terms of values at t. For further reference see Kamien et al. (1991).

(9) and (10) imply[5] that

$$I_t = \begin{Bmatrix} 0 \\ ? \\ B_t \end{Bmatrix} \quad \text{when} \quad m_t - \rho - 2\sigma I_t \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} 0 \tag{11}$$

Moreover, it must hold for the optimal solution, that

$$\partial\mathcal{H} / \partial I \partial I = -2\sigma \leq 0 \tag{12}$$

which is true as $\sigma > 0$,

$$\text{and} \quad \lim_{t \to \infty} e^{-rt}\mathcal{H} = 0 \tag{13}$$

(13) stands for the transversality condition.

Solution of optimization problem

The solution of the theoretical model shows us the optimal paths of the capital, investment and the farmer's supply in the conditions where the farmer is not financially constrained and where she/he faces a financial constraint, i.e. credit rationing occurs in our case. Then, the dynamic analysis is based on the solution.

As the control variable I_t is bounded, the farmer might face the following three situations according to (11):

A) $m_t < \rho + 2\sigma I_t \quad \Rightarrow \quad I_t = 0$ – if the marginal value of a unit of capital is less than its marginal cost, then no investment is carried out. This means that the capital decreases by the rate of physical depreciation *b*.

No investment might also be carried out in the situation when $B_t = 0$, i.e. in the situation when the farmer has no resources (both internal or external) to carry out the investment.

B) $m_t = \rho + 2\sigma I_t \quad \Rightarrow \quad I_t$ is inside the opened interval $(0; B_t)$, i.e. $0 < I_t < B_t$.

This situation indicates that if the investments are not binding, the farmer follows the well-known rule for capital accumulation, i.e. the farmer chooses the size of investment to equate the marginal value of a unit of capital and marginal costs.

C) $m_t > \rho + 2\sigma I_t \quad \Rightarrow \quad I_t = B_t$

The marginal value *m* of a unit of capital is higher than its marginal costs. It follows that the farmer is not able to raise the investment at time *t* as much as she/he would like (following the rule $m_t = \rho + 2\sigma I_t$) to equate marginal values. This situation represents the occurrence of credit rationing. That is, the farmer is financially constrained because she/he has no additional resources to finance the required level of investment. The theoretical analysis and numerical application are focused on this situation.

Ad C) The situation with financial constraints (credit rationing) – solution

In this part, we show the solution to the situation when the investment is binding, i.e. the farmer faces credit rationing.

The solution to situation C follows the following proposition 1. Since the optimization problem is autonomous and the horizon is infinite, we inquire about a stationary state (see the following definition).

Definition 1: *A stationary state (or steady state) is the state, in which* $k' = I' = m' = 0$.

The volume of *k*, *I* and *m* in the stationary state is denoted by k_S, I_S and m_S.

Proposition 1: *If the farmer's investment is bounded (financially constrained) in time* t_i *and* $k_i < k_s$ *then: the stationary state is approached by selecting* $I_t = B_t$ *on the interval* $t_i \leq t < t_j$, *in which the farmer is financially constrained, and* $I_t = \frac{m_t - \rho}{2\sigma}$ *on the interval* $t_j \leq t < t_k$, *in which she/he is not constrained, if the farmer follows the optimal path of capital given by* $m_t = \rho + 2\sigma I_t$.

Moreover, as B_t stands for the upper bound of the investment at time *t* and is assumed to be generated by the occurrence of credit rationing in the credit market, we assume that B_t is a function of the equity. Subsequently, we may approximate the increase in the equity and, thus, the increase in the potential collateral (which in fact increases the upper bound of investment) by the constant increase in B_t for some interval, i.e. $dB_t = (s)dt$, which means that $B_t = B_0 + s \times t$, $t \in (0, t_n)$. If the conditions in the equity change significantly, then the linear function changes as well.

Then, since the investment at time t_0 is the largest and then is decreasing, we assume that the investment is bounded on the first part of the optimization horizon, i.e. we assume that without loss of generality the following hypothesis holds.

Hypothesis for the problem solution: *to find the solution we assume that the optimization horizon* $t \in [0; \infty)$, *during which the system reaches the stationary state* ***(if it is feasible)****, can be divided into 2 parts:*

(i) $0 \leq t < t_1$ – *in this period the farmer is financially constrained and, thus, the investment* I_t *is equal to the upper bound* B_t.

(ii) $t_1 \leq t < \infty$ – *from time* t_1 *the farmer is not financially constrained, i.e. the farmer's investment is inside the interval* $0 < I_t < B_t$.

[5]Conditions (9) and (10) are equivalent to (11).

The time t_1 is determined by the condition:

$$w_{2t} = m_t - \rho - 2\sigma I_t = 0 \tag{14}$$

or equivalently

$$I_t = \frac{m_t - \rho}{2\sigma} \tag{15}$$

To sum up, according to *proposition 1* and the above stated *hypothesis* the optimization horizon is divided into two parts: with and without financial constraints. The switching point from the first to the second period is given by (15) which can be used for finding the switching time t_1. The solutions to the optimization problem for period 1 and 2 are as follows.

Period 1 – The situation with financial constraints (credit rationing) – solution

To find the solution for the period, in which the farmer is financially constrained, we solve our optimization problem according to *the proposition (1)* (part (i)) and *the hypothesis* for the problem solution. This means that when we substitute in the necessary condition (8) B_t for I_t, it results in (16). As the investment is bounded in this case, we get the solution to our optimization problem with the financial constraint by solving (16), i.e. we only solve the first order linear differential equation

$$k_t' = B_t - bk_t \tag{16}$$

The solution to (16) is

$$k_t = \frac{B_0}{b} + \frac{s \times t}{b} - \frac{s}{b^2} + e^{-bt} \times c_1 \tag{17}$$

where c_1 is the constant of integration, which can be determined knowing that $k(0) = k_0$ (see condition (3)). Thus, (17) can be rewritten

$$k_{\bullet t}^{\bullet} = \frac{B_0}{b} + \frac{s \times t}{b} - \frac{s}{b^2} + e^{-bt} \times (k_0 - \frac{B_0}{b} + \frac{s}{b^2}) \tag{17'}$$

Equation (17) ' gives the optimal path of the capital when the farmer is financially constraint. Optimal path of I_t is given by B_t, i.e. $B_t = B_0 + s \times t$, see the above given assumption.

The multiplier w_{2t} can be expressed by (18). The multiplier stands for the difference between the shadow price of the capital and its marginal costs.

$$w_{2t} = m_t - \rho - 2\sigma I_t \tag{18}$$

where $I_t = B_t$ and m_t is determined according to (7) with the capital given in (17)'. The equation m_t is then used to determine the switching time t_1. In fact, (18) becomes our switching condition (15) when w_{2t} is equal to 0.

Period 2 – The situation without financial constraints (credit rationing) – solution

According to the hypothesis investment is not binding in the second period. That is, we face a standard dynamic nonlinear optimization problem. However, for the needs of the total solution to situation C and the following scenario analysis, the solution is closely exposed.

The necessary conditions (7)–(9) can be reduced to two ordinary differential equations in the variables (k, m) or (k, I). For the need of our analysis we choose the second possibility. That is, we solve two ordinary differential equations in the variables k and I. To get two differential equations in (k, I), we need to differentiate equation (9) with respect to time (note that if investment is not binding, then the multipliers w_{1t} and w_{2t} are equal to zero) to get:

$$-2\sigma I_t' + m_t' = 0 \tag{19}$$

Then, we may substitute (7) for m' in (19) that yields (20).

$$-2\sigma I_t' + m_t(r+b) - p\alpha\beta k_t^{\beta-1} z = 0 \tag{20}$$

Finally, we may eliminate m_t by the substitution $m_t = \rho + 2\sigma I_t$ from the equation (9). Solving for I_t' and together with (8) we get the system of ordinary nonlinear differential equations.

$$I_t' = \frac{(r+b)(\rho + 2\sigma I_t) - p\alpha\beta k_t^{\beta-1} z}{2\sigma} \tag{21}$$

and

$$k_t' = I_t - bk_t \tag{22}$$

The optimal paths $k_t^{\bullet}$ and $I_t^{\bullet}$ given time independent parameters (α, β, p, z, b, r, ρ, σ) satisfy (21) and (22).

To find the optimal paths of the capital and investment, we start with the stationary state solution of (21) and (22). According to *definition 1*, the stationary state solution is the simultaneous solution of (21) and (22) when $k_t' = I_t' = 0$. That is, we may get k_S and I_S if we solve the following pair of algebraic equations.

$$0 = \frac{(r+b)(\rho + 2\sigma I_t) - p\alpha\beta\, k_t^{\beta-1} z}{2\sigma} \tag{23}$$

$$0 = I_t - bk_t \tag{24}$$

which result in

$$I_S = \frac{p\alpha\beta\, k_S^{\beta-1} z}{(r+b)2\sigma} - \frac{\rho}{2\sigma} \tag{25}$$

$$k_S = \frac{I_S}{b} \tag{26}$$

Second, we need to examine the local stability of the stationary solution. As the system of the differential equations is nonlinear, we use the Jacobian matrix to deduce the local stability of the system in the stationary state, i.e. we evaluate it at $k_t' = I_t' = 0$.

The Jacobian matrix for our problem is as follows:

$$J_f(k_S, I_S) \overset{def}{=} \left.\begin{bmatrix} \frac{\partial k'}{\partial k} & \frac{\partial k'}{\partial I} \\ \frac{\partial I'}{\partial k} & \frac{\partial I'}{\partial I} \end{bmatrix}\right|_{\substack{k'=0 \\ I'=0}} =$$

$$= \begin{bmatrix} -b & 1 \\ \frac{-p\alpha\beta(\beta-1)k_S^{\beta-2}z}{2\sigma} & (r+b) \end{bmatrix} \qquad (27)$$

From (27), $tr[J_t(k_s, I_s)] = r > 0$, it follows that the sum of eigenvalues of $J_f(k_s, I_s)$ is equal to the discount rate r. Thus, the eigenvalues do not have both negative real parts, which suggest that the stationary state is not locally stable. But (as shown in Caputo, 2005), since $\lim_{t\to\infty} e^{-}$, $k_t^\bullet \to k_S$ and $m_t \to m_S$ as $t \to \infty$, if all admissible paths k_t are bounded, or $\lim_{t\to\infty} k_t$ exists for all admissible paths, then this means that $\lim_{t\to\infty} e^{-rt} m\left[k_t^\bullet - k_t\right] = 0$ and the limiting transversality condition is satisfied. Then $k_t^\bullet$ and $I_t^\bullet$ are a solution of the model. This means that a solution exists. The existence of the solution also immediately follows from the fact $\left|J_f(k_S, I_S\right| < 0$, which is evident from (28).

$$\left|J_f(k_S, I_S)\right| = \left.\left[\underbrace{\frac{\partial k'}{\partial k}}_{-} \cdot \underbrace{\frac{\partial I'}{\partial I}}_{+} - \underbrace{\frac{\partial k'}{\partial I}}_{+} \cdot \underbrace{\frac{\partial I'}{\partial k}}_{+}\right]\right|_{\substack{k'=0 \\ I'=0}} < 0 \qquad (28)$$

Therefore, at least one path to the stationary state must exist. Since $\left|J_f(k_S, I_S)\right| < 0$, the eigenvalues are real and have opposite signs. And since their sum is equal to r, the larger one must be positive. Thus, the Routh-Hurwitz condition is satisfied. Hence, the eigenvalues are real and of opposite sign, the stationary point is a saddlepoint, which is reached by two trajectories.

As the system of the differential equations (21) and (22) is nonlinear, it is difficult to find the explicit solution to this system. However, to be able to analyze and simulate the paths of capital and investment into the stationary state we may use the method of linearization of the system in the neighborhood of the stationary state. Thus, we use Taylor's theorem to linearize our system of nonlinear ordinary differential equations in the neighborhood of the stationary state. Because we assume that the higher order terms are small, the linearized system of (21) and (22) is as follows:

$$\begin{bmatrix} \Delta k' \\ \Delta I' \end{bmatrix} = \begin{bmatrix} -b & 1 \\ j_{21} & (r+b) \end{bmatrix} \begin{bmatrix} k_t - k_S \\ I_t - I_S \end{bmatrix} \qquad (29)$$

where

$$j_{21} = \left.\frac{\partial I'}{\partial k}\right|_{\substack{k'=0 \\ I'=0}} = \frac{-p\alpha\beta(\beta-1)k_S^{\beta-2}z}{2\sigma}$$

$\Delta k' = k'$ and $\Delta I' = I'$.

With theorem 25.1 of Simon and Blume (1994), (see Caputo, 2005), the general solution of (29) can be found by

$$\begin{bmatrix} k_t - k_S \\ I_t - I_S \end{bmatrix} = c_1 v^1 e^{\gamma_1 t} + c_2 v^2 e^{\gamma_2 t} \qquad (30)$$

where c_1 and c_2 are constants of integration, v^1 and $v^2 \in \Re^2$ are eigenvectors of $J_f(k_s, I_s)$ corresponding to eigenvalues γ_1 and γ_2.

Finding v^1, v^2, c_1 and c_2, we may write the specific solution to the linearized system of differential equations (29), which describes the optimal path of the capital and investment to the stationary state, as follows:[6]

$$\begin{bmatrix} k_t^\bullet \\ I_t^\bullet \end{bmatrix} = \begin{bmatrix} k_S \\ I_S \end{bmatrix} + \left[k_0 - k_S\right] \times \begin{bmatrix} 1 \\ \frac{j_{21}}{\gamma_1 - r - b} \end{bmatrix} \times e^{\gamma_1 t} \qquad (31)$$

where γ_1 states for the negative eigenvalue.

Finally, we may define the path for the farmer's supply. From the definition of the problem it is evident that the optimal path of farmer's supply is given by

$$y_t^\bullet = \alpha k_t^{\bullet\beta} z \qquad (31)'$$

To sum up, the optimal paths of the capital and investment under financial constraints which are the subject of the study of the next section, are given by the equations (17)', $I_t = B_t = B_0 + s \times t$ and (31) (with the capital k_0 that is equal to the size of the capital in switching time t_1) and with the switching condition (15).

Firstly, the solution can be graphically represented by using the k–I plane[7]. To develop the k–I plane we need to find k and I nullclines or $k' = 0$ and $I' = 0$ isoclines, respectively. The $k' = 0$ isocline is derived from the equation (22), in which we let $k' = 0$. Thus the $k' = 0$ isocline is equal to

[6]For further reference see Caputo (2005).

[7]Or equivalently the k–m plane can be used.

$$0 = I_t - bk_t, \text{ i.e. } I_t = bk_t \tag{32}$$

Now, we may consider $k' = 0$ isocline, that is, points satisfying (32). As (32) is a simple linear function without an intercept, we may conclude that the isocline is a straight line, which goes through the origin and increases with slope b. It is easy to verify that above the locus k' is positive, i.e. k is increasing, and vice versa (see Figure 1).

Next, the $I' = 0$ isocline results from (21) by letting $I' = 0$, i.e. we have

$$(r+b)(\rho + 2\sigma I_t) = p\alpha\beta k_t^{\beta-1} z \tag{33}$$

We can verify the slope of $I' = 0$ isocline based on the definition of the production function. Since the production function is concave, the second derivative is negative. This implies that the isocline is increasing. Above the $I' = 0$ locus, $I'$is positive and vice versa (see again Figure 1).

From the derivation of isoclines it is evident that the intersection of the isoclines represents the fixed point of the system. The isoclines partition the phase plane into isosectors in which the trajectories of the system are monotonic as depicted in Figure 1. Moreover, Figure 1 shows the dynamics of the system and two optimal trajectories, i.e. trajectories leading to the stationary state. However, the optimal trajectory leading from left to right is feasible only if the farmer is able to carry out such an investment. In other words, it represents the optimal path when the farmer is not financially constrained. If the farmer is financially constrained, then she/he follows the solution in period 1 and 2. That is, the farmer chooses the investment in the size of the upper bound B_t. As the upper bound increases with increased capital or collateral, respectively, which we approximated by a constant increase during the time, it can be depicted by an increasing convex function B_t as a function of $k_{\bullet t}^{\bullet}$ in the Figure 1 (see the bold line). Then, the optimal path of the farmer with financial constraints is given by the bold line, $B_t = bk_{\bullet t}^{\bullet} + s/b - e^{-bt}b(k_0 - B_0/b + s/b^2)$, till the intersection with the optimal path without financial constraints. The farmer is not financially constrained from the intersection and follows the optimal path derived in period 2.

It can also happen that the stationary state is not feasible for the farmer. It is the situation when the farmer's financial constraint is very tough that the bold line does not intersect the optimal path. More precisely, the bold line lies below the stationary state. Thus, the farmer is not able to reach her/his economic equilibrium (of course in this case B_t is not a linear function of time, i.e. $B_t = B_0 + s \times t$, in the period $t \in [0, \infty)$).

Figure 2a shows the paths of capital for both farmers with and without financial constraint. The farmer without financial constraint follows the solid line from the beginning. The farmer with financial constraint follows the dashed line till time t_1.[8] From time t_1 she/he also follows the solid line, which is below the solid line of the farmer without financial constraint. From the Figure it is evident that both paths have the same limit, i.e. both farmers reach the stationary value of capital. But the farmer who faces the financial constraints reaches the saddle point later. Figures 2b and 2c show the paths of investment and also the farmers' supplies. The depiction implicitly

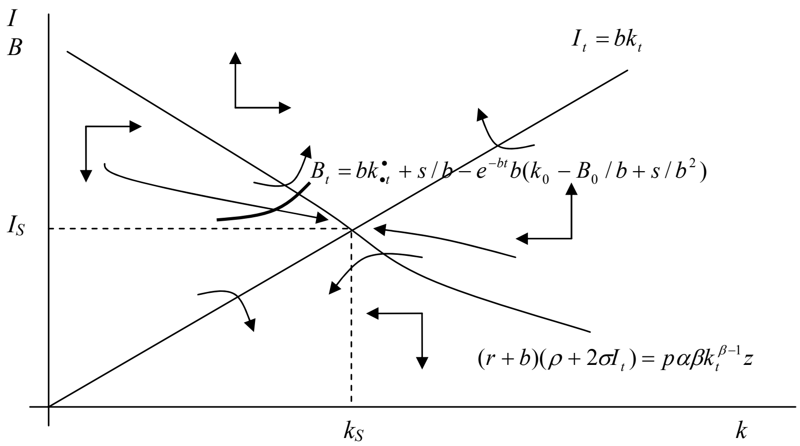

Figure 1. k–I plane

Source: author's depiction based on Kamien et al. (1991)

[8]The dashed line is depicted as a straight line in Figures 2a, c for the simplicity of depiction even if it has in fact also a concave shape in these two Figures.

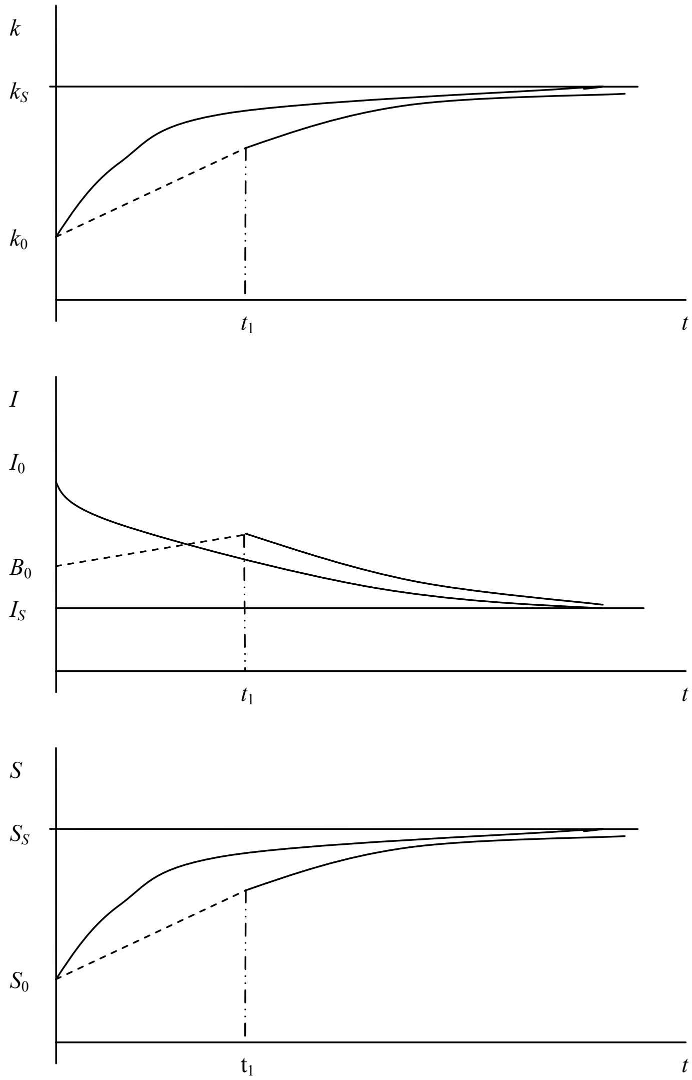

Figure 2a, b, c. The time paths of the capital, investment and farmer's supply

Source: author's depiction

assumes that the stationary state is feasible for the financially constrained farmer.

CONCLUSION

The derived theoretical model shows that the occurrence of credit rationing or in general financial constraints, respectively, significantly determines a farmer's economic equilibrium. That is, the farmer, who faces credit rationing, is for some time outside her/his economic equilibrium. Consequently, she/he reaches the saddle point later (if it is feasible) compared to the non-credit constrained farmer. This results to the loss of production that is equal to the space between the paths with and without financial constraints. To sum up, we may conclude that, in general, the hypothesis of the paper holds.

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