# Evaluating the knowledge, relevance and experience of expert decision makers utilizing the Fuzzy-AHP

## *Použití Fuzzy-AHP pro hodnocení znalostí, relevantnosti a zkušeností expertů*

S. Aly, I. Vrana

*Department of Information Engineering, Faculty of Economics and Management, Czech University of Life Sciences, Prague, Czech Republic*

**Abstract:** The reliance on experts' decision is considered a very efficient solution especially in case of ill-structured and vague decision making situations. This necessitates being able to identify the expert or experts group whose decision must be particularly assigned a greater weight or importance. This paper is concerned with a method of how to determine the weights of a group of expert decision makers, taking into account the vagueness associated with such evaluation.

**Key words:** Group Decision Making, Analytical Hierarchy Process (AHP), Fuzzy-AHP, experts judgments, ill-structured decision making, decision making weights/importance's

**Abstrakt:** Spolehnout se na stanovisko expertů je velmi účinný přístup zejména pro rozhodování ve špatně strukturovaných a vágních situacích. To ovšem vyžaduje schopnost rozpoznat experty nebo skupiny expertů, jejichž stanovisku je možné přisoudit velkou váhu či význam. Článek se zabývá metodou, jak určit váhy pro skupinu expertů s uvažováním neurčitosti takového posouzení.

**Klíčová slova:** skupinové rozhodování, analytický hierarchický proces AHP, Fuzzy-AHP, posudky expertů, špatně strukturovaný rozhodovací problém, váhy při rozhodování

In ill-structured decision situations, it is often difficult to evaluate the correct decision solution beforehand; that is unless some influential future events occur (see e.g. Svoboda 2008, 2007; Michalski 2008; Franěk at al. 2007). In such situations, usually not all affecting variables are known; secondly, some of the known variables are usually stochastic, vague, and qualitative in nature. This is why this decision-making environment necessitates the reliance on multiple experts' opinions, in order to enhance the quality of the obtained decision solution. Crucial and necessary to be associated with those opinions are the weights, which are important to distinguish the information that may assist in resolving the potential conflict among those multiple judgments, and to provide information for differences in the experts‘competencies.

In many Group Decision Making (GDM) contexts and applications, it is not always valid that all group members' or expert decision makers have equal importances with respect to the decision being made or to the particularity of the decision- making transaction. This is because the degree of relevancy, knowledge, and experience may not be equal among those experts or decision makers. Therefore, there must be an allowance for such differences in weights or importances. Over and above, most of the real world GDM takes place in an ambiguous decision situation, in which the value of inputs as well as pertinent data and the sequences of the possible actions are vague or not

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precisely known. Therefore, it is very important to manipulate fuzzy values and concepts of evaluation in order to approximate the vague elements of the decision making environment. This is also in order to be comprehensive in taking into account all relevant effects in the solution of the decision-making problem. Within the context of the complex ill-structured problem of evaluating the importances of decision making experts, it is a major prerequisite to be able to handle vagueness and fuzzy measures associated with such decision- making situation. In this paper, the issue of how to reflect objectively the differences in importance's among the participating expert decision makers is considered, and particularly within vague or ambiguous decision making contexts. The main concern is about using a practical, computationally simple and effective approach. Keeping this point in mind, the paper introduces the Fuzzy-AHP as a tool for weighting the importances of experts.

Table 1. The fundamental scale of the AHP importance intensity value

| Importance intensity $a_{ij}$ | Definition |
|---|---|
| 1 | Equal importance of *i* and *j* |
| 2 | Between equal and weak importance of *i* over *j* |
| 3 | Weak importance of of *i* over *j* |
| 4 | Between weak and strong importance of *i* over *j* |
| 5 | Strong importance of *i* over *j* |
| 6 | Between strong and demonstrated importance of *i* over *j* |
| 7 | Demonstrated importance of *i* over *j* |
| 8 | Between demonstrated and absolute importance of *i* over *j* |
| 9 | Absolute importance of *i* over *j* |

## THE AHP

The analytic hierarchy process (AHP) developed by Saaty (1980) is a decision-making tool that can handle unstructured or semi-structured decisions with multi-person and multi-criteria inputs. It is a decision model that relaxes the measurement of related factors to subjective managerial inputs on multiple criteria. The AHP has several advantages, including its acceptance of inconsistencies in managerial judgments/perceptions and its user friendliness because users may directly input judgment data without requiring further mathematical knowledge. It also allows users to structure complex problems in the form of a hierarchy or a set of integrated levels. The AHP can also be combined with well-known techniques of operation research to handle more difficult problems. One of the main advantages of this method is the relative ease, with which it handles multiple criteria. In addition to this, the AHP is easier to understand and can effectively handle both qualitative and quantitative data. The use of the AHP does not involve cumbersome mathematics. The AHP involves the principles of decomposition, pair wise comparisons, and priority vector generation and synthesis (Duran, Aguilo 2008). Saaty uses the eigenvector method to determine the relative weights among the various criteria based on the pair-wise comparison matrix, positive reciprocal matrix $A = [a_{ij}]$. Table 1 gives the scales of intensity importance used to compare alternatives and criteria.

Saaty defined $\lambda_{max}$ as the largest eigen vector of the matrix *A*, and the weight $w_i$ as a component of the normalized eigen vector corresponding to, $\lambda_{max}$, where:

$$w_i = r_i / (r_1 + r_2 + ... + r_n) \quad (1)$$

and $r_i$ is the geometric mean of each row:

$$r_i = \left( \prod_{j=1}^{n} a_{ij} \right)^{1/n} \quad (2)$$

Saaty's approach has provided with the capability to assess the consistency of the assigned relative importances in the pair-wise comparison matrix. This can be done through computing the consistency ratio (*CR*) from the consistency index (*CI*) and the random index (*RI*), as follows:

$$CR = CI/RI \quad (3)$$

$$CI = \frac{\lambda_{max} - n}{n - 1} \quad (4)$$

where:
*n* = the number of compared criteria, alternatives, or factors

Table 2. The random index *RI* versus the number of assessed factors *n*

| *n* | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *RI* | 0 | 0 | 0.58 | 0.9 | 1.12 | 1.24 | 1.32 | 1.41 | 1.45 | 1.49 | 1.51 | 1.58 |

The values of $RI$ for different values of $n$ is given by the Table 2.

The use of $RI$ is a basic element of the AHP and this table (used in all books) was determined experimentally. Its meaning is as follows: the random index $RI$ is the average consistency index of 100 randomly generated (inconsistent) pairwise comparisons matrices.

Though the purpose of the AHP is to capture the expert's knowledge, the conventional AHP still cannot reflect the human thinking style. In spite of its popularity, this method is often criticized because of a series of pitfalls associated with the AHP technique, which can be summarized as follows (Duran, Aguilo 2008):

– Its inability to adequately handle the inherent uncertainty and imprecision associated with the mapping of the decision-maker's perception to exact numbers (Lefley, Sarkis 1997).
– In the traditional formulation of the AHP, human judgments are represented as exact (or crisp, according to the fuzzy logic terminology) numbers. However, in many practical cases the human preference model is uncertain and decision-makers might be reluctant or unable to assign the exact numerical values to the comparison judgments.
– Although the use of the discrete scale of 1–9 has the advantage of simplicity, the AHP does not take into account the uncertainty associated with the mapping of one's judgment to a number.

Given the aforementioned limitations, one solution is the use of fuzzy set theory to allow incorporating unquantifiable, incomplete and partially known information into the AHP decision model. Therefore, in this paper, a fuzzy extension of the AHP; that is Fuzzy-AHP is proposed to enable considering vagueness associated with evaluating relative importances of decision making experts. In the next sections, the basic elements and procedures of the Fuzzy-AHP technique is described, and an application of the proposed technique for the evaluation of knowledge, experience and relevance of the expert decision makers is introduced and illustrated by an example.

## FUZZY-AHP

The Fuzzy-AHP extends the Saaty's AHP by combining it with the fuzzy set theory. In Laarhoven, Pedrycz (1983) and Boender et al. (1989) a fuzzy version of the Saaty's AHP method was developed. In that version of fuzzy AHP, triangular fuzzy numbers were used with pair-wise comparisons in order to compute the weights of importance of the decision criteria. Thus, all elements in the judgment matrix and weight vectors are represented by triangular fuzzy numbers. A fuzzy number $\widetilde{A}$ expresses the meaning 'about A'. For fuzzy numbers we use triangular fuzzy numbers (that is, fuzzy numbers with lower ($l$), modal ($m$), and upper ($u$) values), because they are simpler than trapezoidal fuzzy numbers. A fuzzy triangular number is defined as follows:

DEFINITION (Dubois, Prade 1980): A fuzzy number $M$ on $R$ is defined to be a fuzzy triangular number if its membership function $\mu_m : R \rightarrow [0,1]$ is equal to:

$$\mu_m = \begin{matrix} \frac{1}{m-l}x - \frac{l}{m-l} & x = \in[l,m] \\ \frac{1}{m-u}x - \frac{l}{m-u} & x = \in[m,u] \\ 0 & \text{otherwise} \end{matrix} \tag{5}$$

Where $l \leq m \leq u$, and $l$ and $u$ stand for the lower and upper values of the support of the fuzzy number $M$, respectively, and $m$ for the modal value. A fuzzy triangular number, as expressed by Equation (5), will be denoted by ($l, m, u$). Fuzzy membership function and the definition of a fuzzy number are shown in Figure 1.

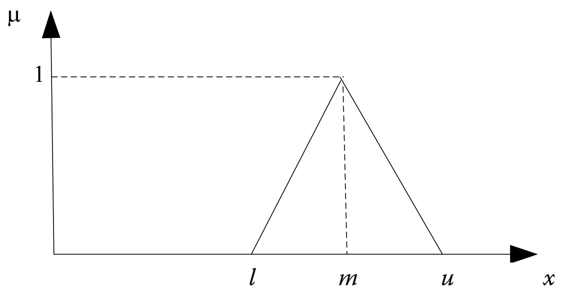


Figure1. The membership of a fuzzy triangular number

Some basic relevant operations on fuzzy triangular numbers, which were developed and used in Laarhoven and Pedrycz (1983), Triantaphyllou and Lin (1996), are defined as follows. For any two fuzzy triangular numbers $\widetilde{A} = (a_1, a_2, a_3)$, $\widetilde{B} = (b_1, b_2, b_3)$:

$\widetilde{A} \oplus \widetilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$ *for addition*
$\widetilde{A} \otimes \widetilde{B} = (a_1 \times b_1, a_2 \times b_2, a_3 \times b_3)$ *for multiplication*
$\widetilde{A} / \widetilde{B} = (a_1/b_3, a_2/b_2, a_3/b_1)$ *for division*
$1/\widetilde{A} = (1/a_3, 1/a_2, 1/a_1)$ *for reciprocal*
$(\widetilde{A})^n = (a_1^n, a_2^n, a_3^n)$ *for power*

Therefore, using fuzzy triangular numbers, the decision-maker, facing a complex and uncertain problem, can express his/her comparison judgments as uncertain ratios, such as 'about two times more important', 'between two and four times less important'.

To accomplish this, the standard AHP steps were extended to incorporate the operations on the fuzzy triangular numbers. Thus, fuzzy judgment matrices are built using fuzzy triangular numbers instead of crisp numbers as was in the AHP. Recently several approaches (Laarhoven and Pedrycz 1983; Buckley 1985; Boender et al. 1989; Chang 1996) have been proposed to extend the AHP into Fuzzy-AHP. In this paper, we adopt the simpler and most transparent and direct procedure, which is described as follows:

**Step 1** – Using the fuzzy judgment scale given in Table 3, adopted from Dagdeviren and Yuksel (2008) scale operations on triangular fuzzy numbers, a **fuzzy positive reciprocal matrix is constructed**:

$$\tilde{A} = \left[\tilde{a}_{ij}\right] \tag{6}$$

and the geometric mean of each row are computed $\tilde{a}_{ij} = (l_{ij}, m_{ij}\ u_{ij})$:

$$\tilde{r} = \sqrt[n]{\prod_{j=1}^{n} \tilde{a}_{ij}} \tag{7}$$

Then the normalized weight $\tilde{w}_i$ is determined using the following formula (Dagdeviren, Yuksel 2008):

$$\tilde{w}_i = \left( \frac{l_i}{\sum_{i=1}^{n} u_i}, \frac{m_i}{\sum_{i=1}^{n} m_i}, \frac{u_i}{\sum_{i=1}^{n} l_i} \right) \tag{8}$$

**Step 2 – Consistency check:** The decision maker has to redo the ratios when the comparison matrix fails to pass the consistency test. The value of $\lambda_{max}$ is computed as the modal value of the resulting fuzzy number, through employing the original Saaty's procedure, but using the operations on fuzzy triangular numbers. Then, if the $CR > 0.1$, then the fuzzy judgment matrix must be revised until reaching or being below 0.1 consistency value.

**Step 3 – Synthesizing**: the weights obtained for each alternative under each criterion is first multiplied respectively to the weight of the criterion, and then summed up over all criteria to obtain the final weight for each alternative. The same procedure can be utilized between each two subsequent level in the Fuzzy-AHP evaluation hierarchy.

**Step 4 – Ranking:** Given the final weights of alternatives, expressed in fuzzy triangular form, now we need to defuzzify these fuzzy weights in order to be able rank alternatives in a non-fuzzy forms (here in this paper alternatives are the evaluated experts' importances). This could be accomplished through utilizing the Best Non-fuzzy Performance BNP values defuzzification method (Chen et al. 2008). Thus, the overall judgmental priorities of experts opinions can be now determined using defuzzification by the BNP, since a fuzzy number represents the fuzzy synthetic decision reached for each alternative, we need to defuzzify these fuzzy numbers in order to compare the alternatives ranking method. In previous works, the procedure of defuzzification has been to locate the best non-fuzzy performance (BNP) value, based on the center of area (COA). The COA is a simple and practical method, and there is no need to introduce the preferences of any evaluators. The COA method's BNP value for triangular fuzzy performance score can be calculated as follows:

$$BNP = l + \frac{(u-l)+(m-l)}{3} \tag{9}$$

Next section, the proposed Fuzzy-AHP is applied to the evaluation of experts' importances.

## EVALUATING EXPERTS' DECISION MAKING IMPORTANCES

Suppose that the company's chief executive officer (CEO), considered as a supra decision maker, who

Table 3. Linguistic scales for intensity importance

| Linguistic scale for importance | Triangular fuzzy scale | Triangular fuzzy reciprocal scale |
|---|---|---|
| Just equal | (1, 1, 1) | (1, 1, 1) |
| Equally important (EI) | (1/2, 1, 3/2) | (2/3, 1, 2) |
| Weakly more important (WMI) | (1, 3/2, 2) | (1/2, 2/3, 1) |
| Strongly more important (SMI) | (3/2, 2, 5/2) | (2/5, 1/2, 2/3) |
| Very strongly more important (VSMI) | (2, 5/2, 3) | (1/3, 2/5, 1/2) |
| Absolutely more important (AMI) | (5/2, 3, 7/2) | (2/7, 1/3, 2/5) |

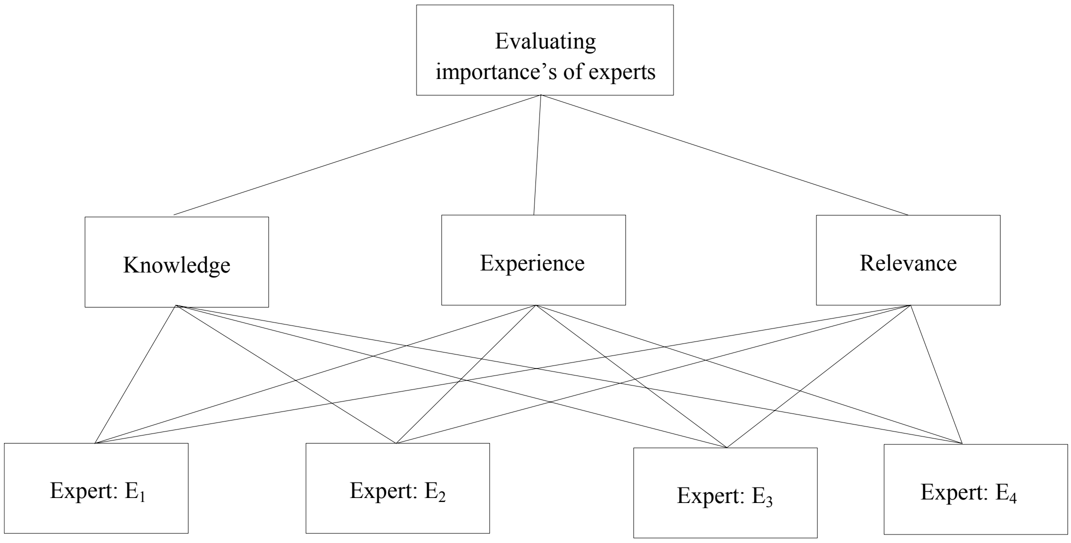


Figure 2. Experts' weights evaluation hierarchy

has a sufficient knowledge and experience to assess the relative importance of every experts, will assign the relative importance intensity values in pair-wise comparisons using the fuzzy linguistic scale (Table 3). Three criteria are considered to be related to the assessing experts' decision making capabilities, and are to be included as basic parameters in comparisons:

- Knowledge: the amount of important knowledge and information each expert bears.
- Experience: the age and historical deepness of the expertise contained in each expert.
- Relevance: the degree of how much each expert has knowledge pertaining and relating to the decision problem.

Then, following the AHP hierarchy shown in Figure 2, the supra decision maker is to conduct all the needed pair-wise comparisons judgments using the fuzzified scale in Table 3. Suppose that his judgments and matrices were as in Tables 4–7. The computations and the associated consistency ratios are then presented below.

Having computed the weights of each expert under each judgment criteria, these weights are then synthesized for all criteria to give the final weights of each expert as a whole. Table 8 shows these synthesizing computations.

Thus in order to rank the experts' final weights utilizing crisp values, the final fuzzy triangular weights shown in Table 8 are defuzzified using equation (9) to compute the best non-fuzzy performance value (BNP), and gives:

$BNP_{E_1} = 0.22$ $BNP_{E_2} = 0.26$ $BNP_{E_3} = 0.29$

$BNP_{E_4} = 0.36$

Table 4. Comparison matrix of the importance of the three judgment criteria

| | Knowledge | Experience | Relevance | Weights |
|---|---|---|---|---|
| Knowledge | (1, 1, 1) | (2/3, 1, 2) | (2/5, 1/2, 2/3) | (0.17, 0.26, 0.29) |
| Experience | (1/2, 1, 3/2) | (1, 1, 1) | (2/3, 1, 2) | (0.18, 0.33, 0.47) |
| Relevance | (3/2, 2, 5/2) | (1/2, 1, 3/2) | (1, 1, 1) | (0.24, 0.41, 0.51) |

CR = 0.046 < 0.1 (Acceptable)

Table 5. Comparison matrix of the four experts with respect to knowledge criterion

| | $E_1$ | $E_2$ | $E_3$ | $E_4$ | Weights |
|---|---|---|---|---|---|
| $E_1$ | (1, 1, 1) | (2/3, 1, 2) | (2/5, 1/2, 2/3) | (2, 5/2, 3) | (0.16, 0.24, 0.43) |
| $E_2$ | (1/2, 1, 3/2) | (1, 1, 1) | (1/2, 2/3, 1) | (3/2, 2, 5/2) | (0.01, 0.25, 0.43) |
| $E_3$ | (3/2, 2, 5/2) | (1, 3/2, 2) | (1, 1, 1) | (5/2, 3, 7/2) | (0.24, 0.4, 0.63) |
| $E_4$ | (1/3, 2/5, 1/2) | (2/5, 1/2, 2/3) | (2/7, 1/3, 2/5) | (1, 1, 1) | (0.08, 0.12, 0.19) |

CR = 0.009 < 0.1 (Acceptable)

Table 6. Comparison matrix of the four experts with respect to experience criterion

| | $E_1$ | $E_2$ | $E_3$ | $E_4$ | Weights |
|---|---|---|---|---|---|
| $E_1$ | (1, 1, 1) | (1/3, 2/5, 1/2) | (1, 3/2, 2) | (2/7, 1/3, 2/5) | (0.1, 0.15, 0.21) |
| $E_2$ | (2, 5/2, 3) | (1, 1, 1) | (5/2, 3, 7/2) | (1/2, 2/3, 1) | (0.23, 0.33, 0.47) |
| $E_3$ | (1/2, 2/3, 1) | (2/7, 1/3, 2/5) | (1, 1, 1) | (2/7, 1/3, 2/5) | (0.08, 0.11, 0.16) |
| $E_4$ | (5/2, 3, 7/2) | (1, 3/2, 2) | (5/2, 3, 7/2) | (1, 1, 1) | (0.29, 0.42, 0.58) |

CR = 0.015 < 0.1 (Acceptable)

Table 7. Comparison matrix of the four experts with respect to relevance criterion

| | $E_1$ | $E_2$ | $E_3$ | $E_4$ | Weights |
|---|---|---|---|---|---|
| $E_1$ | (1, 1, 1) | (3/2, 2, 5/2) | (1/2, 1, 3/2) | (2/7, 1/3, 2/5) | (0.12, 0.21, 0.34) |
| $E_2$ | (2/5, 1/2, 2/3) | (1, 1, 1) | (1/2, 2/3, 1) | (2/5, 1/2, 2/3) | (0.1, 0.15, 0.26) |
| $E_3$ | (2/3, 1, 2) | (1, 3/2, 2) | (1, 1, 1) | (2/3, 1, 2) | (0.15, 0.26, 0.53) |
| $E_4$ | (5/2, 3, 7/2) | (3/2, 2, 5/2) | (1/2, 1, 3/2) | (1, 1, 1) | (0.21, 0.37, 0.59) |

CR = 0.056 < 0.1 (Acceptable)

Table 8. Synthesizing experts' weights respect to the evaluation criteria

| | Knowledge (0.17, 0.26, 0.29) | Experience (0.18, 0.33, 0.47) | Relevance (0.24, 0.41, 0.51) | Final weights |
|---|---|---|---|---|
| $E_1$ | (0.16, 0.24, 0.43) | (0.1, 0.15, 0.21) | (0.12, 0.21, 0.34) | (0.07, 0.2, 0.4) |
| $E_2$ | (0.01, 0.25, 0.43) | (0.23, 0.33, 0.47) | (0.1, 0.15, 0.26) | (0.07, 0.24, 0.48) |
| $E_3$ | (0.24, 0.4, 0.63) | (0.08, 0.11, 0.16) | (0.15, 0.26, 0.53) | (0.09, 0.25, 0.53) |
| $E_4$ | (0.08, 0.12, 0.19) | (0.29, 0.42, 0.58) | (0.21, 0.37, 0.59) | (0.12, 0.32, 0.63) |

Then, these values could be normalized into, 0.2, 0.23, 0.25, & 0.32 to sum up 1. Clearly the opinion of the expert number four should be deemed the most important. However, it is obvious in the result of this example that relatively the importances or weights of these experts are close to each other, but anyway, there is still a little difference that could be useful in resolving the potential conflict.

## CONCLUSION

The problem of evaluating the importances of the expert decision makers in conducting a critical decision making is extremely important because the resulting weights can serve as an important tool for the conflict resolution, a typical case and frequent in complex ill-structured situations. The importance's evaluation itself is ill-structured, because factors determining importances are all not known and mainly are subjective. The extension of the efficient AHP into Fuzzy-AHP enables handling the vagueness associated with subjective assignments and comparisons. The merit of using a fuzzy approach is to assign the relative importance of attributes using fuzzy numbers instead of crisp numbers. Finally, the adopted procedure of the Fuzzy-AHP in this paper is simple and computationally not cumbersome.

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*Contact address:*

Shady Aly, Ivan Vrana, Czech University of of Life Sciences in Prague, Kamýcká 129, 169 51 Prague 6-Suchdol, Czech Republic
e-mail: vrana@pef.czu.cz, shady@pef.czu.cz