Conception of the model of agriculture with production and non-production function

Koncipování modelu zemědělství s produkční a mimoprodukční funkcí

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Abstract: The article deals with an optimisation of the relation between production and non-production functions of agriculture. In the model, a two-product production process is presupposed, the result of which is agricultural production and a complex of other effects arising in agriculture. To obtain them, two aggregated inputs – capital and labour – are used. To maximise the summary economic result, the procedure of derivation of the optimal extent of both production processes proceeding as well as non-production effects in minimisation of both inputs in their summary is worked out in the article. In the article, the knowledge of the research gained in framework of solution of the institutional research intention MSM: 411100013 "Efficient integration of the Czech agrarian sector into the frame of the European structures – presumption of sustainable development" is included.

Key word: production and non-production function, model of agriculture, production process, agricultural production

Abstrakt: Článek se zabývá optimalizací vztahu meziprodukčního a mimoprodukčního poslání zemědělství. V modelu se předpokládá dvouproduktový výrobní proces, jehož výsledkem je zemědělská produkce a souhrn všech ostatních efektů vznikajících v zemědělství. K jejich získání jsou vynaloženy dva agregované vstupy – kapitál a práce. K maximalizaci souhrnného ekonomického výsledku je v článku rozpracován postup odvození optimálního rozsahu obou výrobních procesů poskytujících jak produkční, tak mimoprodukční efekty při minimalizaci obou vstupů v jejich souhrnu. V článku jsou zahrnuty poznatky z výzkumu získané v rámci řešení institucionálního výzkumného záměru MSM: 411100013 „Efektivní integrace českého agrárního sektoru v rámci evropských struktur – předpoklad trvale udržitelného rozvoje".

Klíčová slova: produkční a mimoprodukční funkce, model zemědělství, výrobní proces, zemědělská produkce

INTRODUCTION

In concurrence with increasing agriculture efficiency in countries with developed market economy, also economic power of agricultural businesses increases enabling intensification of reproduction process on farmed land. However, limited absorption land capacity does not enable inputs intensity increase over their limit level resulting partly from biological-technical processes and partly from relatively stable demand for food and consequently for agricultural raw materials. In prominent agricultural businesses which are crucial in use of modern technologies, funds are created with the limited possibility of their allocation into agriculture. Then they have alocation in non-agricultural activities. Production aim of businesses is widened in such way and it includes still more varied spectrum of outputs, many of them is of insubstantial character, from internal economic stimuli. Simultaneously, demand for agricultural raw materials and products of insubstantial character increases which represents external stimuli of multifunctional agriculture rise.

From a viewpoint of steady development conditions, it is purposeful to find a suitable transformation function – a curve of production and non-production possibilities and to use it to set economically motivated relations among these business activities.

AIMS AND METHODOLOGY

Aim of this article is to analyse optimal relations between production mission of agriculture and its non-production activities with a use of inter-branch relation.

The operation hypothesis stems from following assumptions:

– Production processes at a business level are expressed by a production function with aggregated inputs – capital and labour and aggregated outputs.
– Non-production activities in the area of fulfilling of other functions of agriculture, enumeration of which can

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differ in particular businesses, will require at any case analogical procedures for their providing.
- Above-mentioned assumption is valid also in interpreting the landscape-creation function of agriculture in a conception of a combined product as a significant attribute of multifunctional agriculture. Altogether, the landscape-creation function is a standby though still more important result of production activities of agriculture. From the viewpoint of aggregated agricultural output respecting all its processes, it is an intermediate product.
- Also other outputs of multifunctional agriculture, as well as of an insubstantial character, will require capital and labour otherwise it would mean disturbance of the homeostasis principle.
- By aggregation of business outputs the multifunctional character of the branch as a whole is formed.

In such a concurrence for solving of the mentioned aim, it is possible to use principles of production modelling. Production is within its frame represented by models in which one or two inputs are used to create one output, or a complex of particular inputs for production of two outputs. These models could be explained simply with charts because it does not need more than 3 axes (x_1, x_2 and y or y_1, y_2 and x) and the resulting chart does not contain more than three dimensions. However, most businesses use many various inputs to produce many various outputs which leads to multifunctional agriculture. Although these models cannot be often diagrammatised, the same maximisation and minimisation rules are valid for them, existing in models factor-product, factor-factor and product-product. In the next part of the article, there is mentioned an attempt to formulate some general rules which are valid for businesses which use many inputs to produce many outputs fulfilling the multifunctional mission of agriculture.

RESULTS OF ANALYSIS

Model framing

Suppose that agricultural business uses two inputs, capital (x_1) and labour (x_2), to produce two outputs, agricultural production (y_1) and non-agricultural activities (y_2). An average price of a unit agricultural production is 4 CZK, of non-agricultural activities 8 CZK/hour. Interest rate – capital price is 10% and price of labour unit is 10 CZK. Table 1 show returns and VMP value for each input in production per each output.

Data in the table, although they are useful for the illustration of basic logic of the same marginal return principle, strongly simplify the problem. Marginal product of each unit of capital is supposed to be independent of the availability of the other type of fertiliser. Thus the basic production function for each output represents no interaction between the two inputs and therefore it is additive rather than multiplicative.

Table 1. Two inputs in the production of two outputs[a]

Units of inputs	Agricultural production from capital	VMPx_1y_1	Non-agricultural activities from capital	VMPx_1y_2	Agricultural production from labour inputs	VMPx_2y_1	Non-agricultural activities from labour inputs	VMPx_2y_2
0	70		30		80		20	
		80		40		60		80
1	90		35		95		30	
		60		40		60		64
2	105		40		110		38	
		40		24		40		48
3	115		43		120		44	
		20		16		20		24
4	120		45		125		47	
		8		16		12		8
5	122		47		128		48	
		0		16		8		0
6	122		49		130		48	
		–8		8		4		–8
7	120		50		131		47	
		–8		–8		0		–16
8	118		49		131		45	
		–16		–16		–4		–24
9	114		47		130		42	
		–20		–24		–8		–32
10	109		44		128		38	

[a]The average price of agricultural production is 4 CZK, of non-agricultural activities 8 CZK; capital unit price is 10 CZK

Table 2. Allocation of two inputs for agricultural production and non-agricultural activity

Unit	Input	Output
1	capital	agricultural production
2	labour	non-agricultural activities
3	labour	non-agricultural activities
4	capital	agricultural production
5	labour	agricultural production
6	labour	agricultural production
7	labour	non-agricultural activities
8	capital	agricultural production
9	capital	non-agricultural activities
10	capital	non-agricultural activities

Presume that agricultural enterprise has available only 100 CZK to purchase a total of 10 units of inputs. Table 2 shows how each unit of inputs will be allocated. Units 1 and 2 produce the same VMP as do units 3, 4, and 5, as well as 8, 9, and 10. So it does not matter which unit is allocated as first within the given group.

The general equimarginal return rule requires that

$$p_1 MPPx_1 y_1/v_1 = p_2 MPPx_1 y_2/v_1 = p_1 MPPx_2 y_1/v_2 = \\ = p_2 MPPx_2 y_2/v_2 = K \qquad (1)$$

The VMP of each input in the production of each output will be the same and equal to some number K. The number K is actually a Lagrangian multiplier or an imputed value of additional crown available in case for the purchase of inputs.

In this example, price of both inputs v_1 and v_2 was the same 10 CZK per unit. The last unit of inputs applied in this example produced 40 CZK except the last labour unit applied in non-agricultural activities which produced 48 CZK. The correct allocation would have resulted in the same ration of VMP to the price of the input in production of each output. However, this is often not possible from a tabular data presentation. With the seventh allocation (K in this example would be equal to 4.8), the number K was 4 CZK. The last crown spent on each input gave back 4 CZK in the production of each output.

The general profit-maximization relationship requires that

$$p_1 MPPx_1 y_1/v_1 = p_2 MPPx_1 y_2/v_1 = p_1 MPPx_2 y_1/v_2 = \\ = p_2 MPPx_2 y_2/v_2 = 1 \qquad (2)$$

On the input side

$$MRSx_1 x_2 = v_1/v_2 \qquad (3)$$

on the production of each output. On the output side

$$RPTy_1/y_2 = p_1/p_2 \qquad (4)$$

for each input.

Principles optimisation assuming factor availability

Assume that production of two outputs is governed by two production functions, each with two inputs. Let the production functions be

$$y_1 = h\,(x_{11}, x_{21}) \qquad (5)$$

$$y_2 = j\,(x_{12}, x_{22}) \qquad (6)$$

where y_1 and y_2 determine outputs and h and j are production functions for y_1 and y_2. The first subscript on each x means the input, and the second the product to which it is used. For example, x_{21} is input x_2, which is applied to y_1.

The total amount of x_1 and x_2 are used in the production of y_1 and y_2 are

$$x_1 = x_{11} + x_{12} \qquad (7)$$

$$x_2 = x_{21} + x_{22} \qquad (8)$$

The total revenue from the sale of y_1 and y_2 is

$$R = p_1 y_1 + p_2 y_2 \qquad (9)$$

$$= p_1 h\,(x_{11}, x_{12}) + p_2 j\,(x_{12}, x_{22}) \qquad (10)$$

where p_1 and p_2 are prices y_1 and y_2. The total cost is the sum of the quantities of x_1 and x_2 multiplied by their respective prices.

$$C = v_1 x_1 + v_2 x_2 \qquad (11)$$

$$= v_1\,(x_{11} + x_{12}) + v_2\,(x_{21} + x_{22}) \qquad (12)$$

Profit (Π) is revenue minus costs:

$$\mathrm{P} = R - C$$

$$= p_1 y_1 + p_2 y_1 - v_1 x_1 - v_2 x_2$$

$$= p_1 h(x_{11}, x_{21}) + p_2 j(x_{12}, x_{22}) - v_1(x_{11} + x_{12}) + v_2(x_{21} + x_{22}) \qquad (13)$$

Now let

$$h_1 = \partial h/\partial x_{11} \qquad (14)$$

$$h_2 = \partial h/\partial x_{21} \qquad (15)$$

$$j_1 = \partial j/\partial x_{12} \qquad (16)$$

$$j_2 = \partial j/\partial x_{22} \qquad (17)$$

The first-order conditions for maximum profit entail setting the first derivation of the profit function (13) equal to zero with respect to each input used in the production of each output:

$$\partial \Pi/\partial x_{11} = p_1 h_1 - v_1 = 0 \qquad (18)$$

$$\partial \Pi/\partial x_{21} = p_1 h_2 - v_2 = 0 \qquad (19)$$

$$\partial \Pi/\partial x_{12} = p_2 j_1 - v_1 = 0 \qquad (20)$$

$$\partial \Pi/\partial x_{22} = p_2 j_2 - v_2 = 0 \qquad (21)$$

Equations (18) to (21) can be rearranged in a number of ways. One of them is

$$p_1 h_1/v_1 = p_2 j_1/v_1 = p_1 h_2/v_2 = p_2 j_2/v_2 = 1 \qquad (22)$$

The partial derivation h_1 is the marginal product of x_1 in the production of y_1 or $MPPx_1y_1$; j_1 is the marginal product of x_1 in the production of y_2 or $MPPx_1y_2$; h_2 is the marginal product of x_2 in the production of y_1 or $MPPx_2y_1$; j_2 is the marginal product of x_2 in the production y_2 or $MPPx_2y_2$. Regarding that the equation (22) can be rewritten as:

$$p_1 MPPx_1y_1/v_1 = p_2 MPPx_1y_2/v_1 = p_1 MPPx_2y_1/v_2 = = p_2 MPPx_2y_2/v_2 = 1 \quad (23)$$

Agricultural business should allocate inputs in such a way that the last crown invested in each input in the production of each output returns exactly one crown. The Lagrangian multiplier in the profit maximisation example is 1.

Another way of writing the equations (18) to (21) is:

$$-h_1/h_2 = dx_2/dx_1 = v_1/v_2 \quad \text{in the production of } y_1 \quad (24)$$

$$-j_1/j_2 = dx_2/dx_1 = v_1/v_2 \quad \text{in the production of } y_2 \quad (25)$$

The marginal rate of substitution of x_1 for x_2 must be equal the inverse price ration in the production of both outputs.

Another way of rearranging the equations (18) to (21) is as follows:

$$(p_1h_1/v_1)/(p_2j_1/v_1) = 1 \quad (26)$$

$$(h_1/j_1)(p_1/p_2) = 1 \quad (27)$$

$$j_1/h_1 = p_1/p_2 \quad (28)$$

$$dy_2/dy_1 = p_1/p_2 \quad \text{for input } x_1 \quad (29)$$

$$RPTy_1y_2 = p_1/p_2 \quad \text{for input } x_1 \quad (30)$$

Similarly,

$$j_2/h_2 = p_1/p_2 \quad (31)$$

$$dy_2/dy_1 = p_1/p_2 \quad \text{for input } x_2 \quad (32)$$

$$RPTy_1y_2 = p_1/p_2 \quad \text{for input } x_2 \quad (33)$$

The rate of product transformation must be the same for both inputs in the production of the two outputs and equal the inverse product-price ration.

Of course,

$$h_1 = MPPx_1y_1 = v_1/p_1 \quad (34)$$

$$h_2 = MMPx_2y_1 = v_2/p_1 \quad (35)$$

$$j_1 = MPPx_1y_2 = v_1/p_2 \quad (36)$$

$$j_2 = MPPx_2y_2 = v_2/p_2 \quad (37)$$

In the equations (34) to (37), the marginal product of each input in the production of each output must be equal to the corresponding factor/product price ratio.

Optimisation with limited factor availability

The problem can be also solved within a framework of constrained maximisation. The objective function is the maximisation of revenue subject to the constraint imposed by the availability of resources for the purchase of x_1 and x_2.

The revenue is

$$R = p_1 y_1 + p_2 y_2 = p_1 j\,(x_{12}, x_{22}) \quad (38)$$

The cost is

$$C^\circ = v_1x_{11} + v_1x_{12} + v_2x_{21} + v_2x_{22} \quad (39)$$

All notations are the same as in the section about the general principles. The Lagrangian is

$$L = p_1h_1(x_{11}, x_{21}) + p_2 j\,(x_{12}, x_{22}) + + \lambda\,(C^\circ - v_{11}x_{11} - v_1x_{12} - v_2x_{21} - v_2x_{22}) \quad (40)$$

The corresponding first-order conditions for a constrained revenue maximisation are

$$\partial L/\partial x_{11} = p_1h_1 - \lambda v_1 = 0 \quad (41)$$

$$\partial L/\partial x_{11} = p_1h_2 - \lambda v_1 = 0 \quad (42)$$

$$\partial L/\partial x_{12} = p_2 j_1 - \lambda v_1 = 0 \quad (43)$$

$$\partial L/\partial x_{22} = p_2 j_2 - \lambda v_2 = 0 \quad (44)$$

The equations (41) to (44) can be also rewritten in many ways. For example

$$p_1h_1/v_1 = p_2 j_1/v_1 = p_1h_2/v_2 = p_2 j_2/v_2 = \lambda \quad (45)$$

Partial derivation h_1 is the marginal product of x_1 in the production y_1 or $MPPx_1y_1$; j_1 is the marginal product of x_1 in the production y_2 or $MPPx_1y_2$; h_2 is marginal product of x_2 in the production y_1 or $MPPx_2y_1$; j_2 is marginal product of x_2 in the production y_2 or $MPPx_2y_2$. So the equation (45) can be rewritten as

$$P_1 MPPx_1y_1/v_1 = p_2 MPPx_1y_2/v_1 = p_1 MPPx_2y_1/v_2 = p_2 MPPx_2y_2/v_2 = \lambda \quad (46)$$

The Lagrangian multiplicator λ is the imputed value of an extra crown available for inputs to be used in the production of y_1 and y_2 and allocated in the correct manner. These first-order conditions define a point on both the input and output expansion path.

DISCUSSION

The suggested optimisation procedure is conceived for two aggregated outputs and two aggregated inputs. The applied aggregation enabled a current use of criteria from a common analysis of a branch transformation function as well as from the area of isoquant function analysis. However, the expression of inputs and outputs at this aggregation level cannot be applied sufficiently enough in solution of a concrete resources allocation to secure an optimal extent of multifuntional agriculture including its production function regarding the requirement of average input and output prices derivation. Also in case of qualified estimation which in concurrence with externalities evaluation development reach still more exact results, the set optimal input-output combination has only cognitive – dimensional importance and the internal structure of dis-aggregated inputs and outputs can not be derived directly from it. In case of searching this optimum this suggested procedure can be applied as well but the mathematical solution cannot be illustrated in two-dimensional space of products and factors and criteria of optimality of relations product-product and factor-factor will be valid for "n" outputs and "m" inputs together. Development of computer technique enables such solutions.

CONCLUSION

The category of multifunctional agriculture gains still more cogent content which should respect the principle of cost refund by their benefits. For purposes of this analysis, there is framed a model of optimal relations between production and other functions of agriculture to fulfil the multifunctional mission of agriculture requiring a minimal extent of inputs to secure them. The article supports that by widening the model by an additional equation system also the optimal structure of disaggregated outputs and inputs can be derived. Then the optimal choice of their level leads to the development of competitiveness of relevant business and consequently of the national economy sector.

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