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Crowding out of private stocks by public stocks

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ELECTRONIC SUPPLEMENTARY MATERIAL (ESM):

- Supplementary material S1
- Supplementary table (Table S1)

Supplementary material S1. Public Stocks Crowding-Out Model

Model setting

None of the studies discussed in the previous section developed a conceptual model of the crowding out of private stocks by public stocks. A model is developed using the concept of options and the assumption that public stocks are released when market price exceeds a public stocks release price known by the private market. Options are used because the release of public stocks introduces a discontinuity into the market's price discovery process. Specifically, public stocks augment private market supply only when market price exceeds the public stocks release price. This discontinuity can be modelled as:

$$C_{t,t+n} = \int_{P_{release}}^{\infty} \left(P - P_{release}\right) f_{P_{t,t+n}}\left(P;\mu,\sigma\right) dP \tag{1}$$

where $C_{t,t+n}$ is the value of a call option written at time t for expiration date $t + n$ with a strike price of $P_{release}$, the public stock release price; $P_{t,t+n}$ is the price at time $t + n$ that is unknown at time t with a probability distribution function $f_{P_{t,t+n}}(\cdot;\mu,\sigma)$, which is conventionally assumed to be log-normally distributed; and μ and σ are the location and scale parameters of the distribution function of $P_{t,t+n}$. Value of this call option is the incentive, based on information the market knows at time t, to carry private stocks from time t to time $t + n$ in order to sell at prices higher than $P_{release}$ at time $t + n$.

If the market at time t expects price to exceed $P_{release}$ at time $t + n$ with a non-zero probability, the market must rationally expect any existing public stocks to be released at time $t + n$ with the same non-zero probability equal to $1 - F_{P_{t,t+n}}\left(P_{release};\mu,\sigma\right)$, where $F_{P_{t,t+n}}(\cdot;\mu,\sigma)$ is the cumulative distribution function of $P_{t,t+n}$ and thus $F_{P_{t,t+n}}\left(P_{release};\mu,\sigma\right) < 1$. The resulting expected increase in market supply will cause the market to reduce its expectation that price at time $t + n$ will exceed $P_{release}$, reducing the value of the call option and hence the incentive to keep private stocks in order to sell them at prices above $P_{release}$. Crowding out thus begins once the market assigns a positive probability to the release of public stocks. This usually occurs before public stocks are actually released.

The preceding discussion implies the following extension. If the market expects that $F_{P_{t,t+n}}\left(P_{release};\mu,\sigma\right) = 1$, the call option has no value since the market at time t does not expect price at time $t + n$ to exceed $P_{release}$. Hence, when the market does not expect public stocks to be released, public stocks accumulated for release at $P_{release}$ do not crowd out private stocks.

Because public stocks are released via a government program and not as a response of private firms to a higher market price, it can be conceptualized as a rightward shift of the supply curve along a stationary

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demand curve. Thus, to examine the impact of an expected release of public stocks on private stocks, assume the following inverse demand function:

$$P = P(Q_D) = (\alpha / \beta) - (1/\beta) Q_D + (\varepsilon / \beta) \tag{2}$$

where P is price; Q_D is quantity of demand; α and β are the intercept and the slope of the demand function; and ε is a random variable with mean 0 and variance σ^2. The inverse demand function $P(\cdot):[0,\infty) \rightarrow [0,\infty)$ is, without loss of generality, assumed to be differentiable and negatively sloped throughout, implying each price-quantity combination to be unique. Having $Q_{t,t+n}$ and $Q_{release}$ defined as the quantity of demand associated with $P_{t,t+n}$ and $P_{release}$, respectively, the call option in Equation (1) is transformed with respect to quantities as follows:

$$C_{t,t+n} = \int_0^{Q_{release}} \left(\frac{1}{\beta^2}\right)(Q_{release} - Q) f_{Q_{t,t+n}}(Q;\mu_1,\sigma) dQ \tag{3}$$

Whereas the call option of Equation (1) has value when price is greater than $P_{release}$, the call option of Equation (3) has value when the quantity of private market demand is less than the quantity of private market demand associated with $P_{release}$.

Assuming public stocks are managed so their release does not drive market price below $P_{release}$, release of public stocks of size G adds G to private market supply Q, having impact on the value of the call option in Equation (4) as follows:

$$C_{t,t+n} = \int_0^{Q_{release}-G} (1/\beta^2)\left(Q_{release} - (Q+G)\right) f_{Q_{t,t+n}}(Q;\mu_1,\sigma) dQ \tag{4}$$

Using the Leibniz integration rule, the first derivative of Equation (4) with respect to G is:

$$dC_{t,t+n} / dG = -(1/\beta^2) \int_0^{Q_{release}-G} f_{Q_{t,t+n}}(Q;\mu_1,\sigma) dQ < 0 \tag{5}$$

The higher is the size of public stocks, the lower is the value of this call, assuming the release of public stocks is anticipated. More public stocks reduce the probability that demand will be lower than the demand associated with $P_{release}$ as well as the size of this demand shortfall, i.e. $Q_{release} - Q$. As the call declines in value, incentive to keep private stocks in order to sell them at prices above $P_{release}$ declines as well. In short, private stocks are crowded out as public stocks reduce the potential to profit from price increases. Equation (5) adds new insights to this observation investigated by previous studies. *Ceteris paribus*, the marginal crowding out effect is inversely related to the slope parameter of the demand function. Thus, crowding out can vary by commodity. Second, *ceteris paribus*, the marginal crowding out effect is positively related to the probability that market demand is lower than demand at the public stocks release price (i.e. the probability that market price exceeds $P_{release}$). Thus, crowding out increases as market price approaches $P_{release}$.

Using the Leibniz integration rule to take the derivative of Equation (5) with respect to G [i.e. second derivative of Equation (4) with respect to G] adds other insights:

$$d^2C_{t,t+n} / dG^2 = (1/\beta^2) f_{Q_{t,t+n}}(Q_{release} - G;\mu_1,\sigma) > 0 \tag{6}$$

Ceteris paribus, the rate at which public stocks crowd out private stocks decreases as public stocks increase. The rate of decrease is a function of the probability distribution and slope parameter of the demand function. Thus, the marginal crowding out of private stocks by public stocks is not constant as reported by previous empirical studies. Lastly, if public stocks are large enough to cover all demand shortfalls at $P_{release}$, the call option has no value. Marginal crowding out is zero, implying that adding one unit to public stocks increases total stocks by one unit in this situation.

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Derivation of Equation (3)

Given the assumed inverse demand function in Equation (2), the inverse demand functions associated with $P_{t,t+n}$ and $P_{release}$ are:

$$P_{t,t+n} = \frac{\alpha}{\beta} - \frac{1}{\beta} Q_{t,t+n} + \frac{\varepsilon}{\beta} \tag{11}$$

$$P_{release} = \frac{\alpha}{\beta} - \frac{1}{\beta} Q_{release} + \frac{\varepsilon}{\beta} \tag{12}$$

The probability distribution function of $P_{t,t+n}$ is transformed into the probability distribution function of $Q_{t,t+n}$ as follows:

$$f_{P_{t,t+n}}(P;\mu,\sigma) = f_{P_{t,t+n}}\left(\left\{\frac{\alpha}{\beta} - \frac{1}{\beta} Q + \frac{\varepsilon}{\beta}\right\};\mu,\sigma\right) = f_{Q_{t,t+n}}(Q;\mu_1,\sigma) \tag{13}$$

where $\mu_1 = \mu + \ln(-1/\beta)$, and the support of $f_{Q_{t,t+n}}(Q;\mu_1,\sigma)$ is $Q \in \left[\frac{\alpha+\varepsilon}{\beta},\infty\right)$.

The differential of P in Equation (2) is:

$$dP = -\frac{1}{\beta} dQ \tag{14}$$

Substituting Equations (11–14) into Equation (1) provides:

$$\begin{aligned} C_{t,t+n} &= \int_{P_{release}}^{\infty} (P - P_{release}) f_{P_{t,t+n}}(P;\mu,\sigma) dP \\ &= \int_{P_{release}}^{\infty} \left\{\left[\frac{\alpha}{\beta} - \frac{1}{\beta} Q + \frac{\varepsilon}{\beta}\right] - \left[\frac{\alpha}{\beta} - \frac{1}{\beta} Q_{release} + \frac{\varepsilon}{\beta}\right]\right\} f_{P_{t,t+n}}(P;\mu,\sigma) dP \\ &= \int_{P_{release}}^{\infty} \left\{\left[\frac{\alpha}{\beta} - \frac{1}{\beta} Q + \frac{\varepsilon}{\beta}\right] - \left[\frac{\alpha}{\beta} - \frac{1}{\beta} Q_{release} + \frac{\varepsilon}{\beta}\right]\right\} f_{Q_{t,t+n}}(Q;\mu_1,\sigma) dP \\ &= \int_{P_{release}}^{\infty} \left\{\left[\frac{\alpha}{\beta} - \frac{1}{\beta} Q + \frac{\varepsilon}{\beta}\right] - \left[\frac{\alpha}{\beta} - \frac{1}{\beta} Q_{release} + \frac{\varepsilon}{\beta}\right]\right\} f_{Q_{t,t+n}}(Q;\mu_1,\sigma) \left[-\frac{1}{\beta} dQ\right] \\ &= \int_{P_{release}}^{\infty} \left(-\frac{1}{\beta^2}\right)(Q_{release} - Q) f_{Q_{t,t+n}}(Q;\mu_1,\sigma) dQ \end{aligned} \tag{15}$$

where $\mu_1 = \mu + \ln(-1/\beta)$. The limits of integration in Equation (15) need to be converted from price to quantity terms. This conversion implies the limits run from $Q = Q_{release}$ to $Q = -\infty$. However, it is obvious that $Q > 0$. Using the quantity limits of integration and changing the direction of integration, Equation (15) becomes Equation (3) as follows:

$$C_{t,t+n} = \int_{0}^{Q_{release}} \left(\frac{1}{\beta^2}\right)(Q_{release} - Q) f_{Q_{t,t+n}}(Q;\mu_1,\sigma) dQ \qquad (Q.E.D.) \tag{3}$$

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Derivation of Equation (5)

Given the Leibniz Integration Rule and functions, $h(x,z)$, $a(z)$, and $b(z)$; the following is true:

$$\frac{\partial}{\partial z}\int_{b(z)}^{a(z)} h(x,z)dx = \int_{b(z)}^{a(z)} \frac{\partial h(x,z)}{\partial z}dx + h(a(z),z)\frac{\partial a(z)}{\partial z} - h(b(z),z)\frac{\partial b(z)}{\partial z} \tag{16}$$

Comparing Equations (16) and (4) and letting $x = Q$ and $z = G$, $h(Q,G)$, $a(G)$ and $b(G)$ can be defined as:

$$h(Q,G) = \frac{1}{\beta^2}\left[Q_{release} - (Q+G)\right]f_{Q_{t,t+n}}(Q;\mu_1,\sigma)$$

$$a(G) = Q_{release} - G$$

$$b(G) = 0 \tag{17}$$

The partial derivatives of $h(Q,G)$, $a(G)$, and $b(G)$ with respect to G are:

$$\frac{\partial}{\partial G}h(Q,G) = \frac{\partial}{\partial G}\left\{\frac{1}{\beta^2}\left[Q_{release} - (Q+G)\right]f_{Q_{t,t+n}}(Q;\mu_1,\sigma)\right\} = -\frac{1}{\beta^2}f_{Q_{t,t+n}}(Q;\mu_1,\sigma)$$

$$\frac{\partial}{\partial G}a(G) = \frac{\partial}{\partial G}(Q_{release} - G) = -1$$

$$\frac{\partial}{\partial G}b(G) = \frac{\partial}{\partial G}(0) = 0 \tag{18}$$

We additionally need to find $h(a(G),G)$, which is derived as follows:

$$h(a(G),G) = \frac{1}{\beta^2}\left[Q_{release} - (Q_{release} - G + G)\right]f_{Q_{t,t+n}}(Q_{release} - G;\mu_1,\sigma) = 0 \tag{19}$$

To complete the derivation, substitute Equations (17–19) into the process of taking the partial derivative of Equation (4) with respect to G:

$$\begin{aligned}
\frac{\partial}{\partial G}C_{t,t+n} &= \frac{\partial}{\partial G}\int_0^{Q_{release}-G}(1/\beta^2)\left[Q_{release} - (Q+G)\right]f_{Q_{t,t+n}}(Q;\mu_1,\sigma)dQ \\
&= \frac{\partial}{\partial G}\int_{b(G)}^{a(G)} h(Q,G)dQ \\
&= \int_{b(G)}^{a(G)} \frac{\partial h(Q,G)}{\partial G}dQ + h(a(G),G)\frac{\partial a(G)}{\partial G} - h(b(G),G)\frac{\partial b(G)}{\partial G} \\
&= \int_0^{Q_{release}-G}\left(-\frac{1}{\beta^2}\right)f_{Q_{t,t+n}}(Q;\mu_1,\sigma)dQ + 0 - 0 \\
&= -\frac{1}{\beta^2}\int_0^{Q_{release}-G} f_{Q_{t,t+n}}(Q;\mu_1,\sigma)dQ \quad (Q.E.D.)
\end{aligned} \tag{20}$$

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Derivation of Equation (6)

The components from using the Leibniz Integration rule to take a partial derivative of Equation (5) are:

$$h^*(Q,G) = -\frac{1}{\beta^2} f_{Q_{t,t+n}}(Q;\mu_1,\sigma)$$
$$a^*(G) = Q_{release} - G$$
$$b^*(G) = 0 \quad (21)$$

where * is denoted for new h, a, and b, not for derivatives. The partial derivatives of $h^*(Q,G)$, $a^*(G)$, and $b^*(G)$ with respect to G are:

$$\frac{\partial}{\partial G} h^*(Q,G) = \frac{\partial}{\partial G}\left[-\frac{1}{\beta^2} f_{Q_{t,t+n}}(Q;\mu_1,\sigma)\right] = 0$$
$$\frac{\partial}{\partial G} a^*(G) = \frac{\partial}{\partial G}(Q_{release} - G) = -1$$
$$\frac{\partial}{\partial G} b^*(G) = \frac{\partial}{\partial G}(0) = 0 \quad (22)$$

We additionally need to find $h^*(a^*(G),G)$, which is derived as follows:

$$h^*(a^*(G),G) = -\frac{1}{\beta^2} f_{Q_{t,t+n}}(Q_{release} - G;\mu_1,\sigma) \quad (23)$$

To complete the derivation, substitute Equations (21–23) into the process of taking the partial derivative of Equation (5) with respect to G. All but the second term disappears as follows:

$$\frac{\partial^2}{\partial G^2} C_{t,t+n} = \frac{\partial}{\partial G}\left[-\frac{1}{\beta^2}\int_0^{Q_{release}-G} f_{Q_{t,t+n}}(Q;\mu_1,\sigma)\,dQ\right]$$
$$= \frac{\partial}{\partial G}\int_{b^*(G)}^{a^*(G)} h^*(Q,G)\,dQ$$
$$= \int_{b^*(G)}^{a^*(G)} \frac{\partial h^*(Q,G)}{\partial G}\,dQ + h^*(a^*(G),G)\frac{\partial a^*(G)}{\partial G} - h(b^*(G),G)\frac{\partial b^*(G)}{\partial G}$$
$$= 0 + \left[-\frac{1}{\beta^2} f_{Q_{t,t+n}}(Q_{release} - G;\mu_1,\sigma)\right] \times [-1] + 0$$
$$= \frac{1}{\beta^2} f_{Q_{t,t+n}}(Q_{release} - G;\mu_1,\sigma)\,(Q.E.D) \quad (24)$$

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Supplementary table

Table S1. Descriptive statistics, U.S. corn, soybeans, wheat, 1952–1971 crop years

Variable	Mean	Standard deviation	Min	Max
Corn				
CCC stocks (million bushels)	608	423	97	1327
Private stocks (million bushels)	620	192	347	987
Annualized consumption (million bushels)	3088	720	1949	4189
Public carryout stocks-use ratio (%)	21.91	15.92	2.68	44.27
Private carryout stocks-use ratio (%)	20.06	4.10	13.98	29.09
CBOT September futures price[(A)] (USD/bushel)	1.28	0.15	1.06	1.63
CBOT December futures price[(A)] (USD/bushel)	1.24	0.14	1.04	1.54
Log transformed storage cost-adjusted spread	–0.07	0.03	–0.12	–0.00
U.S. loan rate (USD/bushel)	1.24	0.23	1.03	1.62
Ratio[(B)], December futures price to U.S. loan rate (%)	101.90	18.05	76.23	140.68
Soybeans				
CCC stocks (million bushels)	25	49	0	171
Private stocks (million bushels)	40	47	1	155
Annualized consumption (million bushels)	694	305	278	1258
Public carryout stocks-use ratio (%)	3.85	6.05	0.00	21.75
Private carryout stocks-use ratio (%)	5.82	5.55	0.97	19.73
CBOT September futures price[(A)] (USD/bushel)	2.61	0.40	2.11	3.46
CBOT November futures price[(A)] (USD/bushel)	2.55	0.35	2.14	3.30
Log transformed storage cost-adjusted spread	–0.04	0.03	–0.12	–0.01
U.S. loan rate (USD/bushel)	2.25	0.21	1.85	2.56
Ratio[(B)], November futures price to U.S. loan rate (%)	111.17	16.39	91.97	145.96
Wheat				
CCC stocks (million bushels)	692	382	102	1243
Private stocks (million bushels)	231	181	46	654
Annualized consumption (million bushels)	1231	221	851	1577
Public carryout stocks-use ratio (%)	63.70	38.60	8.50	120.70
Private carryout stocks-use ratio (%)	19.50	14.30	4.80	55.80
KCBOT May futures price[(A)] (USD/bushel)	1.88	0.34	1.33	2.27
KCBOT July futures price[(A)] (USD/bushel)	1.78	0.30	1.32	2.25
Log transformed storage cost-adjusted spread	–0.07	0.06	–0.24	–0.01
U.S. loan rate (USD/bushel)	1.69	0.39	1.25	2.24
Ratio[(B)], July futures price to U.S. loan rate (%)	107.05	12.58	85.51	138.25
U.S. treasury bill rate (3-month)				
Corn and soybeans (% as of August 15)	3.55	1.58	0.93	6.85
Wheat (% as of April 15)	3.41	1.48	1.03	6.38
Physical storage cost[(C)] (cents/bushel)	1.18	0.10	1.10	1.34
Corn	3.66	0.34	3.38	4.13
Soybeans	2.43	0.28	2.19	2.86
Wheat	2.37	0.20	2.19	2.67

[A] futures prices are quoted as of August 15 for corn and soybeans and as of April 15 for wheat; [B] relationship between expected market price and public stock release price; [C] unit for physical storage cost is cents per bushel per three months for corn and per two months for soybeans and wheat; number of observations = 20

Source: original calculation using data from USDA Agricultural Statistics (USDA 1960–1977), Annual Report of the Board of Trade of the City of Chicago (CBOT 1952–1971), Annual Statistical Report of the Board of Trade of Kansas City (KCBOT 1952–1971), Federal Reserve Economic Data of Federal Reserve Bank at St. Louis (2015), and USDA Commodity Credit Corporation (CCC 1979)